力学系数学

6

パラメータ $\mu > 0$ に依存した微分方程式

$$\frac{dx}{dt} = -x^2 + \mu^2$$

を考える．$t = 0$ における初期条件 $x(0) = x_0 \in \mathbb{R}$ を満たす解を $\varphi_t(x_0; \mu)$ と表すとき，以下の問いに答えよ．

(i) 解 $\varphi_t(x_0; \mu)$ を求めよ．

(ii) 任意の $\mu > 0$ と $x_0 \in \mathbb{R}$ に対して，$y = \dfrac{\partial \varphi_t}{\partial x_0}(x_0; \mu)$ が微分方程式

$$\frac{dy}{dt} = -2\varphi_t(x_0; \mu) y$$

の解であることを示せ．

(iii) $t = 0$ における初期条件 $y(0) = y_0 \in \mathbb{R}$ を満足する微分方程式

$$\frac{dy}{dt} = -2\varphi_t(0; 1) y$$

の解を求めよ．

(iv) $t = 0$ における初期条件 $y(0) = y_0 \in \mathbb{R}$ を満足する微分方程式

$$\frac{dy}{dt} = -2\varphi_t(0; 1) y + 2$$

の解を求めよ．

An English Translation:

Mathematics for Dynamical Systems

6

Consider the differential equation

$$\frac{dx}{dt} = -x^2 + \mu^2,$$

which depends on the parameter $\mu > 0$. Let $\varphi_t(x_0; \mu)$ denote the solution satisfying the initial condition $x(0) = x_0 \in \mathbb{R}$ at $t = 0$. Answer the following questions.

(i) Obtain the solution $\varphi_t(x_0; \mu)$.

(ii) Show that $y = \dfrac{\partial \varphi_t}{\partial x_0}(x_0; \mu)$ is a solution to the differential equation

$$\frac{dy}{dt} = -2\varphi_t(x_0; \mu)y$$

for any $\mu > 0$ and $x_0 \in \mathbb{R}$.

(iii) Obtain a solution to the differential equation

$$\frac{dy}{dt} = -2\varphi_t(0; 1)y$$

satisfying the initial condition $y(0) = y_0 \in \mathbb{R}$ at $t = 0$.

(iv) Obtain a solution to the differential equation

$$\frac{dy}{dt} = -2\varphi_t(0; 1)y + 2$$

satisfying the initial condition $y(0) = y_0 \in \mathbb{R}$ at $t = 0$.