Optimal Financial Aid in Environmental Policy: A Real Options Approach*

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Abstract

In this paper, we analyze an environmental policy designed to reduce the emission of pollutants when there is uncertainty over the social costs of environmental damage. We first establish a model that incorporates financial aid in which there are two noncooperative agents: a developing country and a developed country. Then, we derive the closed-form solutions for the optimal levels of financial aid that the developed country gives to the developing country. Then, we compare the social welfare level implied by this model with those implied by two additional models; in one, two agents cooperate, and in the other, agents do not cooperate and there is no financial aid. Hence, we show that the means of providing financial aid plays an important role in reducing the fall in social welfare caused by external effects.

Keywords: external effect; financial aid; social welfare; environmental policy; real options;

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1 Introduction

The Kyoto protocol on global warming came into effect on 16 February 2005. According to this agreement, each country's emissions target must be achieved during the period 2008–2012. For example, Japan will be obliged to reduce its 1990 level of greenhouse gases emissions by 6% (as will Canada, Hungary and Poland), while Switzerland, most central and eastern European states and the European Union will have to reduce their emissions by 8%. However, there are concerns that many developing countries have disregarded this protocol. Hence, the developed countries must investigate feasible solutions as soon as possible.

There has been much research on effective political solutions to environmental problems based on different approaches to environmental economics. For an extensive survey of the literature on environmental policy instruments, see Cropper and Oates (1992). However, almost all of this research applies cost-benefit analysis. Standard cost-benefit analysis cannot simultaneously explain three important characteristics of most of the environmental problems described in Pindyck (2000). First, there is uncertainty about the future costs and benefits of adopting a particular policy. Second, environmental policies designed to reduce ecological damage are typically irreversible; i.e., they are associated with *sunk costs*. Third, policy adoption is rarely a 'now-or-never' proposition. In most cases, it is feasible to delay action and wait for new information.

A real options approach has become a useful tool for evaluating irreversible investment under uncertainty (see Dixit and Pindyck (1994)). Pindyck (2000) and Pindyck (2002) explain how irreversibilities and uncertainty interact to affect the timing and design of policy by using a real options approach. Arrow and Fisher (1974), Henry (1974) and Kolstad (1996) examine the implications of irreversibility and uncertainty for environmental policy from the different point of view. More recent studies have investigated the problem of strategic countries implementing environmental policies (see Barrieu and Chesney (2001) and Ohyama and Tsujimura (2005)). In the strategic framework, a potential problem is the occurrence of an external effect. If one agent implements an environmental policy, this affects the other agent's environment. That is, the implementation of an environmental policy by one agent improves not only that agent's environment, it also improves the environment of the other agent. Barrieu and Chesney (2001) analyzed an environmental problem in the presence of strategic interactions between two agents within a real options framework. Ohyama and Tsujimura (2005) shows that an external effect delays policy implementation. This has crucial implications for international policy and regulation. That is, if each country considers the timing of implementing environmental policy in a noncooperative framework, environmental problems are not resolved optimally.

To resolve this problem, several political measures are available; these include environmental subsidies (see, e.g., Dewees and Sims (1976)), environmental taxes (see Baumol and Oates (1988)), emission trading systems (see, e.g., Maeda (2004) and Montgomery (1972)). Hanley, Shogren and White (1997) and Kolstad (1999) summarize the effectiveness of these political measures on environmental problems. In the context of incorporating an emissions trading system within a real options framework, Insley (2003) examines the optimal decisions of a firm that has the option of retrofitting its plant to reduce pollution and, thereby, of not purchasing emissions allowances. Ohyama and Tsujimura (2006) extends Pindyck (2002) by discussing the optimal values of these political instruments and examines their influence on the behavior of strategic agents. Most studies suggest that the policies mentioned above effectively resolve the problem of external effects. However, it is difficult to maintain an environmental taxation system or an emission trading system in practice because this requires agreement between many countries and firms. Hence, high adjustment costs are expected to accompany the implementation of these systems. Moreover, developing countries cannot be included in such a system. This is because developed and developing countries disagree about what the former have polluted so far.

In this paper, as a second-best solution to the problem of the external effect, we examine the effectiveness of financial aid from a developed country in inducing a developing country to implement its environmental policy more optimally with respect to the overall system. We extend the single-agent model of Pindyck (2000) to the case of two asymmetric agents: the developing country and the developed country. We assume that the developed country is more sensitive to pollution than is the developing country. We show that the external effect and the difference in evaluations of pollution between the two countries imply a level of financial aid that the developed country is willing to give to the developing country. Note, however, that the developing country, as well as the developed country, can behave strategically. Financial aid from a developed country does not necessarily speed up the implementation of environmental policy by a developing country. Taking strategic behavior into account, we derive closed-form optimal levels of financial aid and the optimal timing with which developed and developing countries implement their environmental policies.

It is important to realize that the financial aid framework proposed in this paper is self-motivated, rather than forced. This has the practical implication that the developed country may voluntarily give financial aid to the developing country to induce the developing country to implement its environmental policy sooner. Our numerical examples, which are based on reasonable parameter values, indicate that financial aid benefits developed, as well as developing, countries. Furthermore, we show that providing financial aid is effective in reducing the loss in social welfare that is caused by the external effect. We suggest that the results obtained in this paper can provide additional insights into the role of financial aid in addressing environmental economic problems.

The paper is organized as follows. In Section 2, we describe the model, which is based on that of Pindyck (2000). In Section 3, we extend the model of Pindyck (2000) to a model with two asymmetric agents, which we refer to as the developing country and the developed country. In Section 4, we consider the effects of incorporating financial aid into the two-agent model and derive the optimal level of financial aid. In Section 5, we compare the levels of social welfare from the cooperative model, the noncooperative model and the noncooperative model that incorporates financial aid. Section 6 concludes the paper.

2 A Model of Environmental Policy

In this section, we first describe the model, which is based on those of Dixit and Pindyck (1994) (Chapter 12, Section 3), Pindyck (2002) and Ohyama and Tsujimura (2005). All of these authors consider an environmental economics problem that concerns the optimal timing with which a cost-bearing environmental policy for reducing the flows of a pollutant is adopted. The objective of the underlying agent is to choose the optimal timing that minimizes the expected total environmental damage (in monetary terms) caused by the pollutant.

Suppose that the underlying agent i emits the pollutant at the rate π_t^i, which is the flow of the pollutant over time, and that its stock Y_t^i is governed by

$$dY_t^i = (\pi_t^i - \delta Y_t^i)dt, \quad Y_0^i = y, \tag{1}$$

where δ is the rate of natural decay of the stock of pollutant. It is presumed that the initial stock of the pollutant does not depend on agents' attributes. Let $B^i(X_t, Y_t^i)$ denote agent i's damage (or negative benefit) function associated with the stock of the pollutant Y_t^i. It is assumed that

$$B^i(X_t, Y_t^i) = a^i X_t Y_t^i,$$

where a^i is a constant parameter and X_t is a state variable that reflects changes in tastes and technologies and is assumed to be governed by

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \quad X_0 = x,$$

where $\mu \in \mathbb{R}$ and $\sigma \in \mathbb{R}$ are constants. $(W_t)_{t\geq 0}$ is a standard Brownian motion process. $\mathcal{F}_t$ is generated by W_t in $\mathbb{R}$; i.e., $\mathcal{F}_t = \sigma(W_s, s \leq t)$. Note that X_t is independent of the characteristics of agents.

For simplicity, we assume that π_t^i remains constant at its initial level π_0^i until agent i implements the policy. When agent i implements the policy, π_0^i reduces to π_1^i with $0 \leq \pi_1^i < \pi_0^i$. Thus, equation (1) becomes

$$dY_t^i = \begin{cases} (\pi_0^i - \delta Y_t^i)dt, & 0 \leq t < \tau^i, \\ (\pi_1^i - \delta Y_t^i)dt, & \tau^i \leq t < \infty, \end{cases} \tag{2}$$

where $\tau^i \in \mathbb{T}$ is the (in general, unknown) time that agent i adopts the policy and $\mathbb{T}$ is the class of all implementation times relative to $(\mathcal{F}_t)_{t\geq 0}$. Furthermore, in this paper, we assume that

$$\mathbb{E}\left[\int_0^\infty e^{-r^i t}|B(X_t, Y_t^i)|dt\right] < \infty,$$

where $r^i \in \mathbb{R}_{++}$ is the rate of time preference. Let K^i be the constant cost of implementing the environmental policy for agent i. Therefore, the agent's problem is to choose $\tau^i \in \mathbb{T}$ to minimize the net present value function (see Øksendal (1998)),

$$V^i(x, y) = \inf_{\tau^i \in \mathbb{T}} \mathbb{E}\left[\int_0^\infty e^{-r^i t} B^i(X_t, Y_t^i)dt + e^{-r^i \tau^i} K^i\right],$$

subject to equation (1).

3 The Optimal Timing of Environmental Policy

In this section, we discuss the optimal timing of an environmental policy in the developing and the developed country by extending Ohyama and Tsujimura (2006). We study the case in which there are two competing agents: $i = L, F$. Suppose that both agents have experienced damage from pollutants and wish to determine when they should implement an environmental policy to reduce pollutant emissions. Agent L represents a developed country, and agent F represents a developing country. We assume that if one of the agents (say L) implements the environmental policy, it affects the environment of the other agent (say F). That is, agent F experiences environmental improvement because agent L implements an environmental policy. Barrieu and Chesney (2001) term this effect the *induced effect*. Let $\pi^i_{N_i N_j}$ be the emission flow of agent i, where, for $k \in \{i, j\}$, we specify

$$N_k = \begin{cases} 0, & \text{if agent } k \text{ has not implemented the policy,} \\ 1, & \text{if agent } k \text{ has implemented the policy.} \end{cases}$$

We assume that the total emission reduction does not depend on the magnitude of the external effect. For simplicity, we also assume that the emission structure of each agent is as follows:

$$\begin{aligned} (\pi^L_{00} - \pi^L_{11}) &= (\pi^L_{00} - \pi^L_{10}) + \underbrace{(\pi^F_{00} - \pi^F_{01})}_{\text{external effect}} = (\pi^L_{01} - \pi^L_{11}) + \underbrace{(\pi^F_{10} - \pi^F_{11})}_{\text{external effect}}; \\ (\pi^F_{00} - \pi^F_{11}) &= (\pi^F_{00} - \pi^F_{10}) + \underbrace{(\pi^L_{00} - \pi^L_{01})}_{\text{external effect}} = (\pi^F_{01} - \pi^F_{11}) + \underbrace{(\pi^L_{10} - \pi^L_{11})}_{\text{external effect}}. \end{aligned} \tag{AS.1}$$

In practice, the magnitude of the external effect depends on the type of pollutant under consideration. In our model, the type of underlying pollutant is reflected by the strength of the external effect. For example, suppose that the agents emit greenhouse gases such as CO2 or methane, and that an increase in greenhouse gases is assumed to raise the earth's average temperature. Under these conditions, it is likely that the following equation holds: $\frac{1}{2}(\pi^L_{00} - \pi^L_{11}) = (\pi^L_{00} - \pi^L_{10}) = (\pi^L_{01} - \pi^L_{11})$, $\frac{1}{2}(\pi^F_{00} - \pi^F_{11}) = (\pi^F_{00} - \pi^F_{10}) = (\pi^F_{01} - \pi^F_{11})$. Suppose that the agents emit either SO2 or NOx as primary causes of acid rain. It seems reasonable that the following relationships hold: $\frac{1}{2}(\pi^L_{00} - \pi^L_{11}) < (\pi^L_{00} - \pi^L_{10}) = (\pi^L_{01} - \pi^L_{11})$, $\frac{1}{2}(\pi^F_{00} - \pi^F_{11}) < (\pi^F_{00} - \pi^F_{10}) = (\pi^F_{01} - \pi^F_{11})$. Furthermore, we assume that implementation costs in the developing and developed countries are K^F and K^L, respectively.

Because of the external effect, the timing of the implementation of environmental policies is later when there are two competing agents than when agents cooperate (see Proposition 3.2 of Ohyama and Tsujimura (2006)). This has crucial implications for international policy and regulation. It means that if the implementation of environmental policies is noncooperative, environmental problems are not resolved optimally. In the context of the amount of financial aid, we identify two cases. In both cases, the value functions of both agents are derived. In the first case, the developed country implements the environmental policy first, whereas in the second case, the developing country implements the policy first. The order in which countries implement their policies is influenced by the parameters $r^i, a^i, K^i, k, b, and \pi_1^i - \pi_0^i$, and by the scale of the external effect, where k is the amount of financial aid and b reflects the technology level in the developed country. When the developed country gives financial aid of k to the developing country, it also simultaneously provides technological skills. Therefore, financial aid of k reduces the implementation cost for the developing country by the amount bk. Note that the parameter a^i denotes each country's degree of sensitivity to pollution, which plays an important role in this paper.

3.1 The developing country's problem: the case in which the developed country implements its policy first

We examine the case in which the developed country implements its policy first. The developing country experiences the external effect of the developed country's policy implementation. The dynamics of the pollutant stock, given by equation (2), become

$$dY_t^F = \begin{cases} d\check{Y}_t^F = (\pi_{00}^F - \delta\check{Y}_t^F)dt, & 0 \leq t < \tau_1^L, \\ d\hat{Y}_t^F = (\pi_{01}^F - \delta\hat{Y}_t^F)dt, & \tau_1^L \leq t < \tau_1^F, \\ d\tilde{Y}_t^F = (\pi_{11}^F - \delta\tilde{Y}_t^F)dt, & \tau_1^F \leq t < \infty, \end{cases}$$

where τ_1^L denotes the (generally unknown) time period in which the developed country L adopts the policy. This is defined by

$$\tau_1^L = \inf\{t > 0; X_t \geq x_1^L\}$$

for given x_1^L. τ_1^F represents the (generally unknown) time period in which the developing country F adopts the policy. Note that we assume that $\tau_1^L < \tau_1^F$. The developing country's problem is given by

$$V_1^F(x,y) = \inf_{\tau_1^F \in \mathbb{T}} \mathbb{E}\Bigg[\int_0^{\tau_1^L} e^{-r^F t} B^F(X_t, \check{Y}_t^F) dt + \int_{\tau_1^L}^{\tau_1^F} e^{-r^F t} B^F(X_t, \hat{Y}_t^F) dt \\ + e^{-r^F \tau_1^F}(K^F - bk) + \int_{\tau_1^F}^{\infty} e^{-r^F t} B^F(X_t, \tilde{Y}_t^F) dt \Bigg].$$

Note that the cost of implementing the environmental policy is reduced by the amount of financial aid, which is accompanied by the transfer of technological skills.

By using the strong Markov property and the recursive property of conditional expectations, we obtain the developing country's value function as follows:

$$V_1^F(x,y) = \begin{cases} \frac{a^F x \pi_{00}^F}{(r^F-\mu)(r^F-\mu+\delta)} + \frac{a^F xy}{r^F-\mu+\delta} + \left(\frac{x}{x_1^L}\right)^{\beta^L} \left[-\frac{a^F x_1^L(\pi_{00}^F - \pi_{01}^F)}{(r^F-\mu)(r^F-\mu+\delta)} \right] & \\ \qquad - \left(\frac{x}{x_1^F}\right)^{\beta^F} \left[\frac{a^F x_1^F(\pi_{01}^F - \pi_{11}^F)}{(r^F-\mu)(r^F-\mu+\delta)} - (K^F - bk) \right], & x < x_1^L, \\ \frac{a^F x \pi_{01}^F}{(r^F-\mu)(r^F-\mu+\delta)} + \frac{a^F xy}{r^F-\mu+\delta} - \left(\frac{x}{x_1^F}\right)^{\beta^F} \left[\frac{a^F x_1^F(\pi_{01}^F - \pi_{11}^F)}{(r^F-\mu)(r^F-\mu+\delta)} - (K^F - bk) \right], & x_1^L \le x \le x_1^F, \\ \frac{a^F x \pi_{11}^F}{(r^F-\mu)(r^F-\mu+\delta)} + \frac{a^F xy}{r^F-\mu+\delta} + (K^F - bk), & x \ge x_1^F. \end{cases} \tag{3}$$

where β^i is the positive root of the following characteristic equation: $\frac{1}{2}\sigma^2\beta^i(\beta^i-1)+\mu\beta^i-r^i=0$. For $x_1^L < x < x_1^F$, the sum of the first and second terms on the right-hand side represent the present value of the environmental damage. In this region, because the leader has implemented its environmental policy and because there is an external effect, the follower's emission flow of the pollutant decreases from π_{00}^F to π_{01}^F. The remaining terms represent the value of having the option of implementing the environmental policy in the future. When $x \ge x_1^F$, the sum of the first and second terms continues to represent the cost of environmental damage. Note that the emission flow of the pollutant decreases from π_{01}^F to π_{11}^F because of the implementation of the policy by the follower. The last term represents the policy implementation cost.

The optimal stopping time is

$$\tau_1^F = \inf\{t > 0; X_t \ge x_1^F\},$$

where the threshold x_1^F is determined by the following value-matching and smooth-pasting conditions:

$$x_1^F = \frac{\beta^F}{\beta^F - 1}\left(\frac{(r^F-\mu)(r^F-\mu+\delta)(K^F-bk)}{a^F(\pi_{01}^F-\pi_{11}^F)}\right). \tag{4}$$

3.2 The developing country's problem: the case in which the developing country implements its policy first

We examine the case in which the developing country implements its policy first. The developing country experiences the external effect of the developed country's policy implementation. The dynamics of the pollutant stock, given by equation (2), become

$$dY_t^F = \begin{cases} d\check{Y}_t^F = (\pi_{00}^F - \delta\check{Y}_t^F)dt, & 0 \le t < \tau_2^F, \\ d\hat{Y}_t^F = (\pi_{10}^F - \delta\hat{Y}_t^F)dt, & \tau_2^F \le t < \tau_2^L, \\ d\tilde{Y}_t^F = (\pi_{11}^F - \delta\tilde{Y}_t^F)dt, & \tau_2^L \le t < \infty, \end{cases}$$

where τ_2^L is the (generally unknown) time period in which the developed country L adopts the policy. This is defined by

$$\tau_2^L = \inf\{t > 0; X_t \ge x_2^L\}$$

for given x_2^L. τ_2^F represents the time period in which the developing country implements its policy. Note that we assume that $\tau_2^F < \tau_2^L$. The developing country's problem is solved in the same way as was the developed country's in Subsection 3.1. We obtain the developing country's value function as follows:

$$V_2^F(x,y) = \begin{cases} \frac{a^F x\pi_{00}^F}{(r^F-\mu)(r^F-\mu+\delta)} + \frac{a^F xy}{r^F-\mu+\delta} + \left(\frac{x}{x_2^L}\right)^{\beta^L}\left[-\frac{a^F x_2^L(\pi_{10}^F-\pi_{11}^F)}{(r^F-\mu)(r^F-\mu+\delta)}\right] & \\ \qquad -\left(\frac{x}{x_2^F}\right)^{\beta^F}\left[\frac{a^F x_2^F(\pi_{00}^F-\pi_{10}^F)}{(r^F-\mu)(r^F-\mu+\delta)} - (K^F-bk)\right], & x < x_2^F, \\ \frac{a^F x\pi_{10}^F}{(r^F-\mu)(r^F-\mu+\delta)} + \frac{a^F xy}{r^F-\mu+\delta} + K^F - bk - \left(\frac{x}{x_2^L}\right)^{\beta^L}\left[\frac{a^F x_2^L(\pi_{01}^F-\pi_{11}^F)}{(r^F-\mu)(r^F-\mu+\delta)}\right], & x_2^F \le x \le x_2^L, \\ \frac{a^F x\pi_{11}^F}{(r^F-\mu)(r^F-\mu+\delta)} + \frac{a^F xy}{r^F-\mu+\delta} + K^F - bk, & x \ge x_2^L. \end{cases} \tag{5}$$

The optimal stopping time is

$$\tau_2^F = \inf\{t > 0; X_t \ge x_2^F\},$$

where the threshold x_2^F is determined by the following value-matching and smooth-pasting conditions:

$$x_2^F = \frac{\beta^F}{\beta^F - 1}\left(\frac{(r^F-\mu)(r^F-\mu+\delta)(K^F-bk)}{a^F(\pi_{00}^F-\pi_{10}^F)}\right). \tag{6}$$

Lemma 3.1. *Suppose that assumption (AS.1) holds. Moreover, we assume that*

$$\pi_{00}^F - \pi_{10}^F = \pi_{01}^F - \pi_{11}^F. \tag{AS.2}$$

Then, we obtain

$$x_1^F = x_2^F.$$

Proof. We obtain this result from equations (4) and (6). □

This lemma implies that the timing of the developing country's policy implementation does not depend on which country implements the environmental policy first. In the remainder of this paper, we assume that (AS.2) holds, and we let x^F and τ^F denote $x_1^F = x_2^F$ and $\tau_1^F = \tau_2^F$, respectively.

3.3 The developed country's problem: the case in which the developed country implements its policy first

We examine the case in which the developed country implements its policy first. The developed country experiences the external effect of the developing country's policy implementation. The dynamics of the pollutant stock, given by equation (2), become

$$dY_t^L = \begin{cases} d\check{Y}_t^L = (\pi_{00}^L - \delta\check{Y}_t^L)dt, & 0 \le t < \tau_1^L, \\ d\hat{Y}_t^L = (\pi_{10}^L - \delta\hat{Y}_t^L)dt, & \tau_1^L \le t < \tau^F, \\ d\tilde{Y}_t^L = (\pi_{11}^L - \delta\tilde{Y}_t^L)dt, & \tau^F \le t < \infty, \end{cases}$$

where τ^F is the (generally unknown) time period in which the developing country F adopts the policy, which we derived in Subsection 3.1. The developed country's problem is given by

$$V_1^L(x,y) = \inf_{\tau_1^L\in\mathbb{T}} \mathbb{E}\bigg[\int_0^{\tau_1^L} e^{-rt}B^L(X_t,\check{Y}_t^L)dt + \int_{\tau_1^L}^{\tau^F} e^{-rt}B^L(X_t,\hat{Y}_t^L)dt + e^{-r\tau_1^L}K^L \\ + \int_{\tau^F}^{\infty} e^{-rt}B^L(X_t,\tilde{Y}_t^L)dt + e^{-r\tau^F}k\bigg].$$

Note that the cost of implementing the environmental policy is reduced by the amount of financial aid k, and not by bk. This is because the parameter b denotes the difference in the technological skill levels of the developing country and the developed country. The developed country does not care about the cost of providing technological skills to the developing country.

By using the strong Markov property and the recursive property of conditional expectations, we obtain the developed country's value function as follows:

$$V_1^L(x,y) = \begin{cases} \frac{a^L x \pi_{00}^L}{(r^L-\mu)(r^L-\mu+\delta)} + \frac{a^L xy}{r^L-\mu+\delta} + \left(\frac{x}{x_1^L}\right)^{\beta^L} \left[-\frac{a^L x_1^L(\pi_{00}^L-\pi_{10}^L)}{(r^L-\mu)(r^L-\mu+\delta)} + K^L\right] \\ \quad - \underbrace{\left(\frac{x}{x^F(k)}\right)^{\beta^F} \left[\frac{a^L x^F(k)(\pi_{10}^L - \pi_{11}^L)}{(r^L-\mu)(r^L-\mu+\delta)} - k\right]}_{G(k)}, & x < x_1^L, \\ \frac{a^L x \pi_{10}^L}{(r^L-\mu)(r^L-\mu+\delta)} + \frac{a^L xy}{r^L-\mu+\delta} + K^L - \underbrace{\left(\frac{x}{x^F(k)}\right)^{\beta^F} \left[\frac{a^L x^F(k)(\pi_{10}^L - \pi_{11}^L)}{(r^L-\mu)(r^L-\mu+\delta)} - k\right]}_{G(k)}, & x_1^L \le x \le x^F(k), \\ \frac{a^L x \pi_{11}^L}{(r^L-\mu)(r^L-\mu+\delta)} + \frac{a^L xy}{r^L-\mu+\delta} + (K^L + k), & x \ge x^F(k). \end{cases} \tag{7}$$

The optimal stopping time is

$$\tau_1^L = \inf\{t > 0; X_t \ge x_1^L\},$$

where the threshold x_1^L is determined by the following value-matching and smooth-pasting conditions:

$$x_1^L = \frac{\beta^L}{\beta^L - 1}\left(\frac{(r^L-\mu)(r^L-\mu+\delta)K^L}{a^L(\pi_{00}^L - \pi_{10}^L)}\right). \tag{8}$$

Note that the trigger x_1^L does not depend on the amount of financial aid k. To explain the terms in the value function that can be controlled by the developed country, we define the following:

$$G(k) := \left(\frac{x}{x^F(k)}\right)^{\beta^F} \left[\frac{a^L x^F(k)(\pi_{10}^L - \pi_{11}^L)}{(r^L-\mu)(r^L-\mu+\delta)} - k\right]. \tag{9}$$

Note that because the developed country controls the timing of the developing country's implementation by providing financial aid, the developed country maximizes its value function accordingly.

3.4 The developed country's problem: the case in which the developing country implements its policy first

We examine the case in which the developing country implements its policy first. The developed country experiences the external effect of the developing country's policy implementation. The dynamics of the pollutant stock, given by equation (2), become

$$dY_t^L = \begin{cases} d\check{Y}_t^L = (\pi_{00}^L - \delta \check{Y}_t^L)dt, & 0 \leq t < \tau^F, \\ d\hat{Y}_t^L = (\pi_{01}^L - \delta \hat{Y}_t^L)dt, & \tau^F \leq t < \tau_2^L, \\ d\tilde{Y}_t^L = (\pi_{11}^L - \delta \tilde{Y}_t^L)dt, & \tau_2^L \leq t < \infty, \end{cases}$$

where τ^F is the (generally unknown) time period in which the developing country F adopts the policy derived in Subsection 3.1. As in Subsection 3.3, we obtain the developed country's problem as follows:

$$V_2^L(x,y) = \begin{cases} \frac{a^L x \pi_{00}^L}{(r^L-\mu)(r^L-\mu+\delta)} + \frac{a^L xy}{r^L-\mu+\delta} + \left(\frac{x}{x_2^L}\right)^{\beta^L} \left[-\frac{a^L x_2^L (\pi_{01}^L - \pi_{11}^L)}{(r^L-\mu)(r^L-\mu+\delta)} + K^L\right] \\ \qquad \underbrace{- \left(\frac{x}{x^F(k)}\right)^{\beta^F} \left[\frac{a^L x^F(k)(\pi_{00}^L - \pi_{01}^L)}{(r^L-\mu)(r^L-\mu+\delta)} - k\right]}_{F(k)}, & x < x^F(k), \\ \frac{a^L x \pi_{01}^L}{(r^L-\mu)(r^L-\mu+\delta)} + \frac{a^L xy}{r^L-\mu+\delta} + k + \left(\frac{x}{x_2^L}\right)^{\beta^L} \left[-\frac{a^L x_2^L (\pi_{01}^L - \pi_{11}^L)}{(r^L-\mu)(r^L-\mu+\delta)} + K^L\right], & x^F(k) \leq x \leq x_2^L, \\ \frac{a^L x \pi_{11}^L}{(r^L-\mu)(r^L-\mu+\delta)} + \frac{a^L xy}{r^L-\mu+\delta} + (K^L + k), & x \geq x_2^L. \end{cases} \tag{10}$$

The optimal stopping time is

$$\tau_2^L = \inf\{t > 0; X_t \geq x_2^L\},$$

where the threshold x_2^L is determined by the following value-matching and smooth-pasting conditions:

$$x_2^L = \frac{\beta^L}{\beta^L - 1} \left(\frac{(r^L-\mu)(r^L-\mu+\delta)K^L}{a^L(\pi_{01}^L - \pi_{11}^L)} \right). \tag{11}$$

Note that the trigger x_2^L does not depend on the amount of financial aid k. To clarify the terms in the value function that are controlled by the developed country, we define the following:

$$F(k) := \left(\frac{x}{x^F(k)}\right)^{\beta^F} \left[\frac{a^L x^F(k)(\pi_{00}^L - \pi_{01}^L)}{(r^L-\mu)(r^L-\mu+\delta)} - k\right]. \tag{12}$$

Lemma 3.2. *Suppose that assumptions (AS.1) and (AS.2) hold. Moreover, we assume that*

$$\pi_{00}^L - \pi_{10}^L = \pi_{01}^L - \pi_{11}^L. \tag{AS.3}$$

Then, we obtain

$$x_1^L = x_2^L.$$

Furthermore, this implies

$$G(k) = F(k).$$

Proof. These results follow from equations (8) and (11), and equations (9) and (12). □

This lemma implies that the timing of the developing country's implementation is not influenced by which country implements the environmental policy first. However, this does significantly affect the shape of the value function of the developed country. In the remainder of this paper, we assume that (AS.3) holds, and we let x^L, τ^L and $H(k)$ denote $x_1^L = x_2^L$, $\tau_1^L = \tau_2^L$ and $G(k) = F(k)$, respectively.

4 Optimizing Financial Aid

In this section, we assume that the developed country chooses the optimal financial aid that maximizes its value function. First, we consider the maximization problem without incorporating strategic interactions.

$$\sup_{0 \le k \le \frac{K^F}{b}} H(k)$$

Because

$$\frac{dH(k)}{dk} = \begin{cases} > 0, & k < \frac{QK^F}{1+bQ}, \\ < 0, & \frac{QK^F}{a+bQ} < k \le \frac{K^F}{b}, \end{cases}$$

we obtain the following optimal solution:

$$k^* = \begin{cases} \frac{QK^F}{1+bQ}, & Q > 0, \\ 0, & -\frac{1}{\beta^F b} \le Q \le 0, \end{cases}$$

where

$$\begin{aligned} Q = & \frac{a^L(\pi_{10}^L-\pi_{11}^L)}{a^F(\pi_{01}^F-\pi_{11}^F)} \frac{(r^F-\mu)(r^F-\mu+\delta)}{(r^L-\mu)(r^L-\mu+\delta)} - \frac{1}{\beta^F b} \\ = & \frac{a^L(\pi_{00}^L-\pi_{01}^L)}{a^F(\pi_{00}^F-\pi_{10}^F)} \frac{(r^F-\mu)(r^F-\mu+\delta)}{(r^L-\mu)(r^L-\mu+\delta)} - \frac{1}{\beta^F b}. \end{aligned}$$

If there is no external effect, i.e., if $\pi_{10}^L - \pi_{11}^L = \pi_{00}^L - \pi_{01}^L = 0$, then $Q = -\frac{1}{\beta^F b}$. Under this assumption, the behavior of the developing country is independent of the behavior of the developed country. Consequently, the developed country does not give any financial aid to the developing country.

Definition 4.1. *We define x_3^F and x_4^F as follows:*

$$\begin{aligned} x_3^F &:= x^F(k^*) = \frac{\beta^F}{\beta^F-1}\left(\frac{(r^F-\mu)(r^F-\mu+\delta)(K^F-bk^*)}{a^F(\pi_{01}^F-\pi_{11}^F)}\right) = \frac{\beta^F}{\beta^F-1}\left(\frac{(r^F-\mu)(r^F-\mu+\delta)(K^F-bk^*)}{a^F(\pi_{00}^F-\pi_{10}^F)}\right), \\ x_4^F &:= x^F(0) = \frac{\beta^F}{\beta^F-1}\left(\frac{(r^F-\mu)(r^F-\mu+\delta)K^F}{a^F(\pi_{01}^F-\pi_{11}^F)}\right) = \frac{\beta^F}{\beta^F-1}\left(\frac{(r^F-\mu)(r^F-\mu+\delta)K^F}{a^F(\pi_{00}^F-\pi_{10}^F)}\right). \end{aligned}$$

In this expression, x_3^F is the critical value for the developing country that provides financial aid of k^*, and x_4^F is the critical value for the developing country that receives no financial aid.

We now consider the optimal level of financial aid of a developed country that knows that it will implement the environmental policy before the other country does so. In this case, the developed country minimizes its value function, equation (7), by changing $k \in [0, K^F/b]$. When the value of x is large enough to satisfy $x \geq x_4^F$, $x \geq x^F(k)$ holds for all $k \in [0, K^F/b]$. This implies that the developed country chooses the optimal level $k^{**} = 0$ to minimize the third line of equation(7). On the other hand, when the value of x is small enough to satisfy $x < x_4^F$, the financial aid k that is consistent with $x > x^F(k)$ is not optimal because of the value matching condition at $x = x^F(k)$ in equation (7). Hence, in order to minimize the first and second lines of equation (7), the developed country solves

$$\sup_{\left\{k\in[0,K^F/b] \;\middle|\; x^F(k)\geq x\right\}} H(k) \tag{13}$$

The optimal solution for k^{**} of equation (13) is expressed as

$$k^{**} = \begin{cases} k^*, & 0 < x \leq x_3^F, \\ \frac{K^F}{b} - \frac{x(\beta_1-1)}{\beta_1}\frac{a^F(\pi_{01}^F-\pi_{11}^F)}{b(r^F-\mu)(r^F-\mu+\delta)}, & x_3^F < x < x_4^F. \end{cases}$$

Knowing that it will implement its environmental policy after the other country does so, the developed country minimizes its value function (10) by changing $k \in [0, K^F/b]$. The optimal level of financial aid is k^{**} because $H(k) = G(k) = F(k)$. Thus, the optimal level of financial aid is independent of whether the developed country implements its environmental policy first.

Thus, in a nonstrategic situation, the developed country gives k^{**} and implements its environmental policy at $\tau^L = \inf\{t > 0; X_t \geq x^L\}$, whereas the developing country implements its environmental policy at $\tau^F = \inf\{t > 0; X_t \geq x^{F(k)}\}$. This result applies in a strategic situation in which the developed country may change the threshold and the amount of financial aid in order to reverse the order in which the policies are implemented by comparing V_1^L with V_2^L (or V_1^F with V_2^F in a strategic situation for a developing country).

First, we consider the developing country's strategic decision. We assume that $x^L \leq x^F(k^{**})$, and we let $\tilde{V}_1^F(x, y, z)$ and $\tilde{V}_1^F(x, y, z)$ denote the right-hand side of equation (3) and equation (5), with x^F having been replaced by z, respectively. Then, for $\tilde{x}^F < x^L$, by assumptions (AS.2) and (AS. 3), we have

$$V_1^F(x, y) = \tilde{V}_1^F(x, y, x^F(k^{**})) \leq \tilde{V}_1^F(x, y, x^L) = \tilde{V}_2^F(x, y, x^L) \leq \tilde{V}_2^F(x, y, \tilde{x}^F) \tag{14}$$

Equation (14) shows that the developing country does not change the order of $x^L \leq x^F(k^{**})$ to $\tilde{x}^F < x^L$. Similarly, we can show that the developing country does not reverse the order when $x^L > x^F(k^{**})$.

Next, we investigate whether the developed country changes the order. We assume that $x^L \leq x^F(k^{**})$, and let $\tilde{V}_1^L(x, y, w, k)$ and $\tilde{V}_1^L(x, y, w, k)$ denote the right-hand side of equation (7) and equation (10), with x^L having been replaced by w, respectively. Then, by assumptions (AS.2) and (AS.3), for values of $(\tilde{x}^L, k)$ that satisfy $x^F(k) < \tilde{x}^L$, we have

$$V_1^L(x, y, x^L, k^{**}) \leq \tilde{V}_1^L(x, y, x^F(k), k) = \tilde{V}_2^L(x, y, x^F(k), k) \leq \tilde{V}_2^L(x, y, \tilde{x}^L, k) \tag{15}$$

Equation (15) shows that the developed country does not change the order of $x^L \leq x^F(k^{**})$ to $x^F(k) < \tilde{x}^L$,. Similar results apply when $x^L > x^F(k^{**})$. Therefore, strategic countries make the same decisions as do myopic countries that ignore the possibility of changing the order in which the policies are implemented when assumptions (AS.1) and (AS.2) hold.

Proposition 4.1. *If $x \leq x_3^F$, the optimal level of financial aid is k^*. If $x_3^F < x < x_4^F$, the developed country adjusts its optimal level of financial aid so that the critical value of the developing country is equal to x, as is indicated by $k = \frac{K^F}{b} - \frac{x(\beta_1 - 1)}{\beta_1} \frac{a^F(\pi_{01}^F - \pi_{11}^F)}{b(r^F - \mu)(r^F - \mu + \delta)}$. If $x_4^F \leq x$, the developed country does not offer any financial aid to the developing country.*

What happens if assumptions (AS.1) and (AS.2) are not made? The developing country could attempt to change the order in which policies are implemented. Then, expecting the developing country to renege, the developed country is likely to reconsider the timing and the amount of financial aid. The developing country will consider acting strategically. These actions will be repeated until both countries no longer have an incentive to change their decisions; i.e., the strategies will generate an equilibrium. However, generally, it is difficult to see how such an equilibrium could be reached.

The next proposition concerns the order of policy implementation.

Proposition 4.2. *If*

$$K^L\left(\frac{\beta^L}{\beta^L-1}\right)\frac{(r^L-\mu)(r^L-\mu+\delta)}{a^L(\pi^L_{00}-\pi^L_{10})}\left\{\left(1-\frac{1}{\beta^F}\right)+\frac{a^L b(\pi^L_{10}-\pi^L_{11})}{a^F(\pi^F_{01}-\pi^F_{11})}\frac{(r^F-\mu)(r^F-\mu+\delta)}{(r^L-\mu)(r^L-\mu+\delta)}\right\}$$
$$> K^F\left(\frac{\beta^F}{\beta^F-1}\right)\frac{(r^F-\mu)(r^F-\mu+\delta)}{a^F(\pi^F_{01}-\pi^F_{11})},$$

then $x^F_3 < x^L$.

Proof. This result is obtained by comparing the x^L from Lemma 2.2 with the value of x^F_3 implied by Definition 3.1. □

This inequality depends on the parameters $a^i, b, K^i andr^i$, and on the external effect. According to the proposition, if this inequality holds and $x < x^L$, the developing country implements its environmental policy first.

5 The Social Welfare

In this section, we compare the social welfare level implied by this model with those obtained under two other models; namely the cooperative two-agent model and the noncooperative two-agent model in which there is no financial aid. In the cooperative model, the two agents act as if there is a central commanding agent, because both agents share the same value function. We highlight two possible scenarios that are realistic and that generate interesting insights into improving social welfare. Under Scenario 1, the developed country implements its policy first in all three models. Under Scenario 2, the developing country implements its policy first in the cooperative model and in the noncooperative model in which there is no financial aid. We assume that social welfare in this model is represented by the sum of the value functions of the developing country and the developed country. In addition to (AS.1)–(AS.3), for simplicity, suppose that

$$b = 1,\ r^F = r^L. \tag{AS.4}$$

5.1 Scenario 1

In this scenario, the developed country implements its policy first in all three models.

5.1.1 The cooperative model

When the developing country and the developed country cooperate, social welfare is as follows:

$$SW^C(x,y) = \begin{cases} \frac{a^L x \pi_{00}^L}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x \pi_{00}^F}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} \\ \quad - \left(\frac{x}{x_c^F}\right)^\beta \left[\frac{a^L x_c^F(\pi_{10}^L - \pi_{11}^L)}{\rho_1\rho_2} + \frac{a^F x_c^F(\pi_{01}^F - \pi_{11}^F)}{\rho_1\rho_2} - K^F\right] \\ \quad - \left(\frac{x}{x_c^L}\right)^\beta \left[\frac{a^L x_c^L(\pi_{00}^L - \pi_{10}^L)}{\rho_1\rho_2} + \frac{a^F x_c^L(\pi_{00}^F - \pi_{01}^F)}{\rho_1\rho_2} - K^L\right], & x < x_c^L, \\ \\ \frac{a^L x \pi_{10}^L}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x \pi_{01}^F}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} + K^L \\ \quad - \left(\frac{x}{x_c^F}\right)^\beta \left[\frac{a^L x_c^F(\pi_{10}^L - \pi_{11}^L)}{\rho_1\rho_2} + \frac{a^F x_c^F(\pi_{01}^F - \pi_{11}^F)}{\rho_1\rho_2} - K^F\right], & x_c^L \le x \le x_c^F, \\ \\ \frac{a^L x \pi_{11}^L}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x \pi_{11}^F}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} + K^L + K^F, & x \ge x_c^F, \end{cases} \tag{16}$$

where $\rho_1 = r - \mu$ and $\rho_2 = r + \delta - \mu$. Given the value-matching and smooth-pasting conditions,

$$x_c^F = \frac{\beta}{\beta - 1} \frac{K^F}{\frac{a^F(\pi_{01}^F - \pi_{11}^F)}{\rho_1\rho_2} + \frac{a^L(\pi_{10}^L - \pi_{11}^L)}{\rho_1\rho_2}},$$

$$x_c^L = \frac{\beta}{\beta - 1} \frac{K^L}{\frac{a^L(\pi_{00}^L - \pi_{10}^L)}{\rho_1\rho_2} + \frac{a^F(\pi_{00}^F - \pi_{01}^F)}{\rho_1\rho_2}}.$$

Because there is no external effect in this cooperative model, social welfare is optimal. However, the cooperative model is unrealistic because it implies that all countries agree with the central commanding agent.

5.1.2 The noncooperative model with financial aid: negotiations

When financial aid is introduced into the noncooperative model, social welfare is as follows:

$$SW^{Aid}(x,y) = \begin{cases} \frac{a^L x\pi_{00}^L}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x\pi_{00}^F}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} \\ \quad - \left(\frac{x}{x^F(k^*)}\right)^\beta \left[\frac{a^L x^F(k^*)(\pi_{10}^L - \pi_{11}^L)}{\rho_1\rho_2} + \frac{a^F x^F(k^*)(\pi_{01}^F - \pi_{11}^F)}{\rho_1\rho_2} - (K^F - k^*) - k^*\right] \\ \quad - \left(\frac{x}{x^L}\right)^\beta \left[\frac{a^L x^L(\pi_{00}^L - \pi_{10}^L)}{\rho_1\rho_2} + \frac{a^F x^L(\pi_{00}^F - \pi_{01}^F)}{\rho_1\rho_2} - K^L\right], & x < x^L, \\ \frac{a^L x\pi_{10}^L}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x\pi_{01}^F}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} + K^L \\ \quad - \left(\frac{x}{x^F(k^*)}\right)^\beta \left[a^L \frac{x^F(k^*)(\pi_{10}^L - \pi_{11}^L)}{\rho_1\rho_2} + \frac{a^F x^F(k^*)(\pi_{01}^F - \pi_{11}^F)}{\rho_1\rho_2} - (K^F - k^*) - k^*\right], & x^L \le x \le x_3^F, \\ \frac{a^L x\pi_{10}^L}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x\pi_{01}^F}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} + K^L \\ \quad - \left(\frac{x}{x^F(k)}\right)^\beta \left[\frac{a^L x^F(k)(\pi_{10}^L - \pi_{11}^L)}{\rho_1\rho_2} + \frac{a^F x^F(k)(\pi_{01}^F - \pi_{11}^F)}{\rho_1\rho_2} - (K^F - k) - k\right], & x_3^F \le x \le x_4^F, \\ \frac{a^L x\pi_{11}^L}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x\pi_{11}^F}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} + K^L + K^F, & x \ge x_4^F. \end{cases} \tag{17}$$

In the region of $x_3^F \le x \le x_4^F$, the value of k is consistent with Proposition 4.1. Therefore,

$$k = K^F - \frac{x(\beta-1)}{\beta}\frac{a^F(\pi_{01}^F - \pi_{11}^F)}{\rho_1\rho_2}.$$

Because the developed country can appropriately adjust its financial aid with respect to x, SW^{Aid} is equal to SW^C in the region of $x_3^F \le x \le x_4^F$.

$$x^F(k) = \begin{cases} \underbrace{\frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2(K^F - k^*)}{a^F(\pi_{01}^F - \pi_{11}^F)}\right)}_{=x_3^F}, & x < x_3^F, \\ \frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2(K^F - k)}{a^F(\pi_{01}^F - \pi_{11}^F)}\right), & x_3^F \le x \le x_4^F, \\ \underbrace{\frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2 K^F}{a^F(\pi_{01}^F - \pi_{11}^F)}\right)}_{=x_4^F} & x \ge x_4^F. \end{cases}$$

Note that the critical value in the developing country depends on the level of financial aid. It implies that financial aid plays an important role in this model. Regardless of the amount of financial aid, the critical value in the developed country is given by

$$x^L = \frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2 K^L}{a^L(\pi^L_{00}-\pi^L_{10})}\right).$$

5.1.3 The noncooperative model

Social welfare in the noncooperative model is as follows:

$$SW^N(x,y) = \begin{cases} \frac{a^L x\pi^L_{00}}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x\pi^F_{00}}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} \\ \quad -\left(\frac{x}{x^F_4}\right)^\beta \left[\frac{a^L x^F_4(\pi^L_{10}-\pi^L_{11})}{\rho_1\rho_2} + \frac{a^F x^F_4(\pi^F_{01}-\pi^F_{11})}{\rho_1\rho_2} - K^F\right] \\ \quad -\left(\frac{x}{x^L}\right)^\beta \left[\frac{a^L x^L(\pi^L_{00}-\pi^L_{10})}{\rho_1\rho_2} + \frac{a^F x^L(\pi^F_{00}-\pi^F_{01})}{\rho_1\rho_2} - K^L\right], & x < x^L, \\ \\ \frac{a^L x\pi^L_{10}}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x\pi^F_{01}}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} + K^L \\ \quad -\left(\frac{x}{x^F_4}\right)^\beta \left[\frac{a^L x^F_4(\pi^L_{10}-\pi^L_{11})}{\rho_1\rho_2} + \frac{a^F x^F_4(\pi^F_{01}-\pi^F_{11})}{\rho_1\rho_2} - K^F\right], & x^L \leq x \leq x^F_4, \\ \\ \frac{a^L x\pi^L_{11}}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x\pi^F_{11}}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} + K^L + K^F, & x \geq x^F_4, \end{cases} \tag{18}$$

where

$$x^F_4 = \frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2 K^F}{a^F(\pi^F_{01}-\pi^F_{11})}\right) = \frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2 K^F}{a^F(\pi^F_{00}-\pi^F_{10})}\right),$$

$$x^L = \frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2 K^L}{a^L(\pi^L_{00}-\pi^L_{10})}\right).$$

5.1.4 Comparative results and numerical example

We obtain the following proposition by comparing the social welfare levels implied by the three models; i.e., the cooperative model, the noncooperative model in which there is financial aid and the noncooperative model.

Proposition 5.1. *Suppose that (AS.1)–(AS.4) hold in this scenario. Then, for $0 < x < \infty$, we obtain $SW^C \leq SW^{Aid} \leq SW^N$. Furthermore, $SW^C = SW^{Aid}$ is achieved in the region of $x^F_4 > x \geq x^F_3$. $SW^C = SW^{Aid} = SW^N$ is achieved in the region of $x \geq x^F_4$.*

Proof. The results follow from equations (16),(17) and (18). □

Table 1: Parameter values

Parameter		Value
a^F	Parameter	1
a^L	Parameter	3
r	Discount rate	0.04
δ	Natural rate	0.02
μ	Expected percentage rate of growth of x	0
σ	Volatility parameters of x	0.2
K^F	Cost of implementing the policy for the developing country	\$2.5 billion
K^L	Cost of implementing the policy for the developed country	\$3 billion
π_{00}^F	Initial pollutant flow in the developing country	\$800,000 tons/yr
π_{01}^F	Pollutant flow in the developing country	\$700,000 tons/yr
π_{11}^F	Pollutant flow in the developing country	\$400,000 tons/yr
π_{00}^L	Initial pollutant flow in the developed country	\$500,000 tons/yr
π_{10}^L	Pollutant flow in the developed country	\$200,000 tons/yr
π_{11}^L	Pollutant flow in the developing country	\$100,000 tons/yr

In this subsection, we present numerical examples. Table 1 presents the base-case parameter values used in this scenario. We calculate the thresholds, x_c^L, x^L, x_3^F and x_4^F (see Table 2). These results imply that if the current value of x_0 is \$30 per ton, only the developing country in the noncooperative model does not implement the environmental policy. The developing country in the noncooperative model should adopt the environmental policy when x reaches \$40 per ton.

Figure 1 shows the optimal amount of financial aid with respect to the variable x. In addition, as we explained in the previous section, for $x \in [0, x_3^F]$, the amount of optimal financial aid is k^*; i.e., \$0.83 billion. For $x \in (x_3^F, x_4^F]$, optimal financial aid is appropriately adjusted. For $x \in (x_4^F, \infty)$, the developing country implements the policy immediately without receiving financial aid. Therefore, the developed country need not provide financial aid to the developing country.

Figure 2 has interesting implications. For $0 < x < x_3^F$, $SW^C < SW^{Aid}$ holds for two reasons. First, there is the full external effect of the developed country because the critical value of the developed country is not influenced by financial aid. This implies that the developed country does not implement the environmental policy with the timing implied by the cooperative model. Therefore, around x_c^L, the gap between SW^C and SW^{Aid} increases. Second, the developed country does not voluntarily give unlimited financial aid to the developing country. This means that $x_c^F = x_3^F$ does not always hold. Therefore, around the point x_c^F, the gap of $SW^C - SW^{Aid}$ increases. For $x_3^F < x < x_4^F$, the developed country appropriately adjusts the level of financial aid with respect to x. Figure 2 makes it clear that for $x_3^F < x$, $SW^C = SW^{Aid}$ holds. Thus, the social welfare level in the noncooperative model that incorporates financial aid is optimal for this region. Note that social welfare in the noncooperative model that incorporates financial aid is higher than the level implied by the noncooperative model.

$SW^{Aid} - SW^N$ appears in Figure 3. The line in the upper part of the figure represents the benefit obtained by the developed country from providing financial aid. The gap between the two lines indicates that some of the benefit is shared with the developing country. The point is that the developed country benefits as much as does the developing country.

Table 2: Values of some thresholds

x_c^L	x^L	x_c^F	x_3^F	x_4^F
14.4	16	20	26.667	40

5.2 Scenario 2

In this scenario, the developing country implements its policy first in the cooperative model and in the noncooperative model that incorporates financial aid.

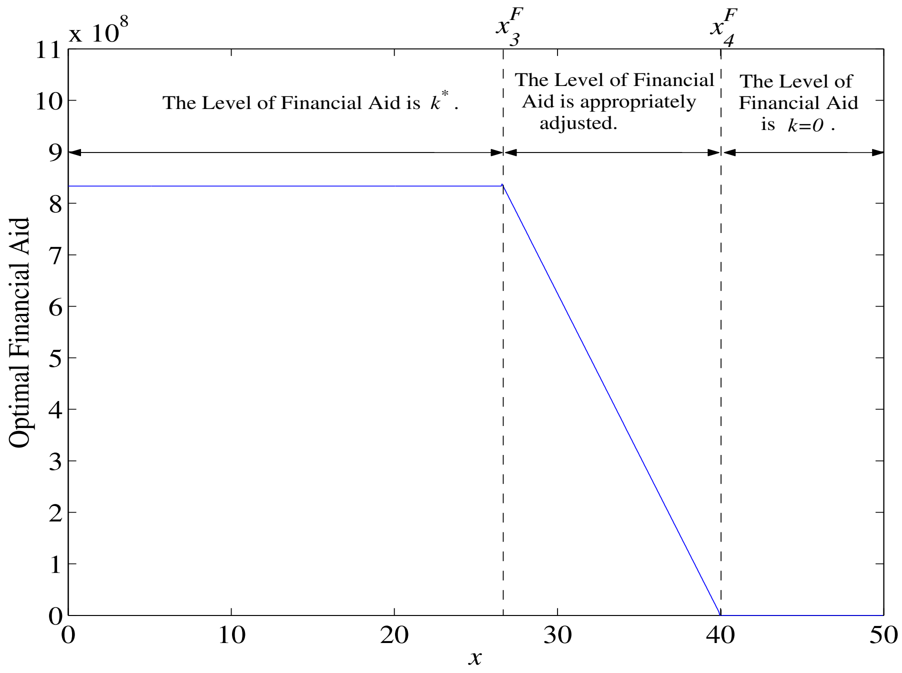

Figure 1: Amount of financial aid.

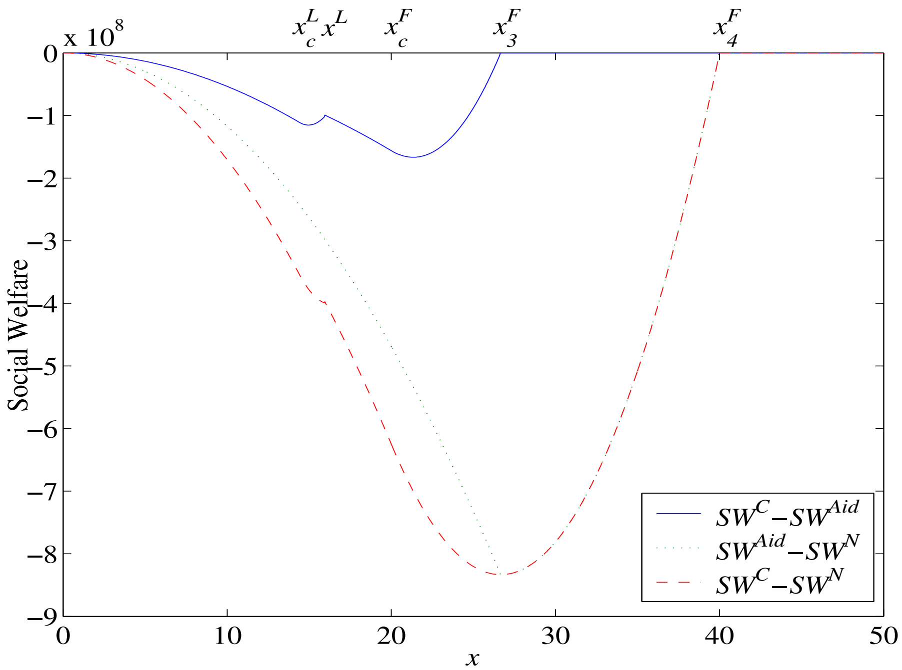

Figure 2: Each social welfare comparison.

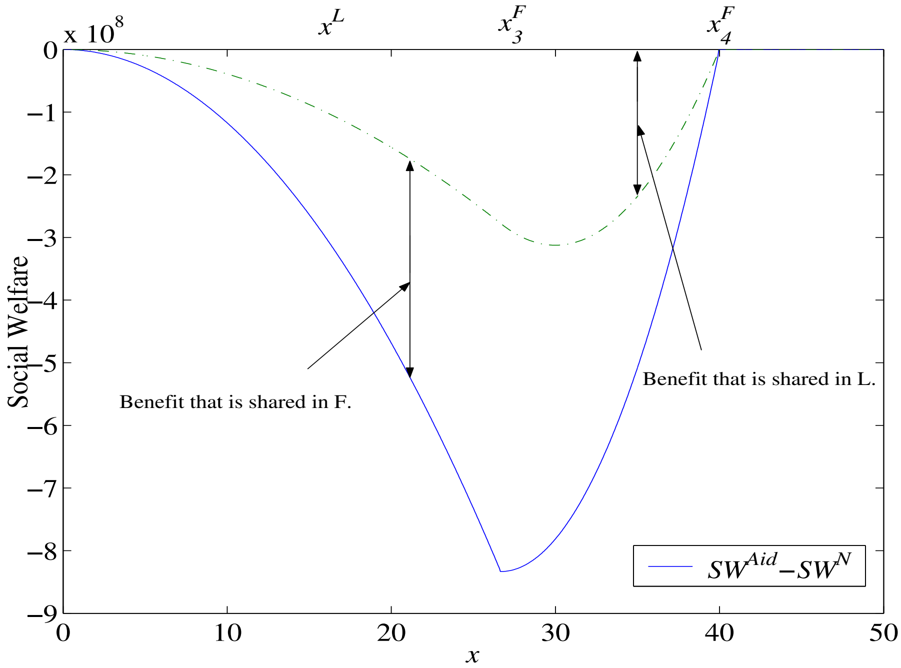

Figure 3: The difference in the social welfare levels SW^{Aid} and SW^{N}. (In addition, this shows the benefit that is shared between the countries)

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5.2.1 The cooperative model

When the developing country and the developed country cooperate, social welfare is as follows:

$$SW^C(x,y) = \begin{cases} \frac{a^L x\pi_{00}^L}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x\pi_{00}^F}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} \\ \quad - \left(\frac{x}{x_c^F}\right)^\beta \left[a^L \frac{x_c^F(\pi_{00}^L - \pi_{01}^L)}{\rho_1\rho_2} + \frac{a^F x_c^F(\pi_{00}^F - \pi_{10}^F)}{\rho_1\rho_2} - K^F\right] \\ \quad - \left(\frac{x}{x_c^L}\right)^\beta \left[\frac{a^L x_c^L(\pi_{01}^L - \pi_{11}^L)}{\rho_1\rho_2} + \frac{a^F x_c^L(\pi_{10}^F - \pi_{11}^F)}{\rho_1\rho_2} - K^L\right], & x < x_c^L, \\ \\ \frac{a^L x\pi_{01}^L}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x\pi_{10}^F}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} + K^L \\ \quad - \left(\frac{x}{x_c^L}\right)^\beta \left[\frac{a^L x_c^L(\pi_{01}^L - \pi_{11}^L)}{\rho_1\rho_2} + \frac{a^F x_c^L(\pi_{10}^F - \pi_{11}^F)}{\rho_1\rho_2} - K^L\right], & x < x_c^L, \\ \\ \frac{a^L x\pi_{11}^L}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x\pi_{11}^F}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} + K^L + K^F, & x \geq x_c^F. \end{cases} \tag{19}$$

The value-matching and smooth-pasting conditions imply

$$x_c^F = \frac{\beta}{\beta - 1} \frac{K^F}{\frac{a^F(\pi_{00}^F - \pi_{10}^F)}{\rho_1\rho_2} + \frac{a^L(\pi_{00}^L - \pi_{01}^L)}{\rho_1\rho_2}},$$

$$x_c^L = \frac{\beta}{\beta - 1} \frac{K^L}{\frac{a^L(\pi_{01}^L - \pi_{11}^L)}{\rho_1\rho_2} + \frac{a^F(\pi_{10}^F - \pi_{11}^F)}{\rho_1\rho_2}}.$$

Because there is no external effect in the cooperative model, social welfare is optimal. However, the cooperative model is unrealistic because it requires agreement between all countries.

5.2.2 The noncooperative model with financial aid: negotiations

When financial aid is introduced into the noncooperative model, social welfare is as follows:

$$SW^{Aid}(x,y) = \begin{cases} \frac{a^L x \pi^L_{00}}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x \pi^F_{00}}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} \\ \quad - \left(\frac{x}{x^F(k^*)}\right)^\beta \left[\frac{a^L x^F(k^*)(\pi^L_{00}-\pi^L_{01})}{\rho_1\rho_2} + \frac{a^F x^F(k^*)(\pi^F_{00}-\pi^F_{10})}{\rho_1\rho_2} - (K^F - k^*) - k^*\right] \\ \quad - \left(\frac{x}{x^L}\right)^\beta \left[\frac{a^L x^L(\pi^L_{01}-\pi^L_{11})}{\rho_1\rho_2} + \frac{a^F x^L(\pi^F_{10}-\pi^F_{11})}{\rho_1\rho_2} - K^L\right], & x < x^F_3, \\ \\ \frac{a^L x \pi^L_{00}}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x \pi^F_{00}}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} \\ \quad - \left(\frac{x}{x^F(k)}\right)^\beta \left[\frac{a^L x^F(k)(\pi^L_{00}-\pi^L_{01})}{\rho_1\rho_2} + \frac{a^F x^F(k)(\pi^F_{00}-\pi^F_{10})}{\rho_1\rho_2} - (K^F - k) - k\right] \\ \quad - \left(\frac{x}{x^L}\right)^\beta \left[\frac{a^L x^L(\pi^L_{01}-\pi^L_{11})}{\rho_1\rho_2} + \frac{a^F x^L(\pi^F_{10}-\pi^F_{11})}{\rho_1\rho_2} - K^L\right], & x^F_3 \le x < x^L, \\ \\ \frac{a^L x \pi^L_{00}}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x \pi^F_{00}}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} + K^L \\ \quad - \left(\frac{x}{x^F(k)}\right)^\beta \left[\frac{a^L x^F(k)(\pi^L_{00}-\pi^L_{01})}{\rho_1\rho_2} + \frac{a^F x^F(k)(\pi^F_{00}-\pi^F_{10})}{\rho_1\rho_2} - (K^F - k) - k\right], & x^L \le x < x^F_4, \\ \\ \frac{a^L x \pi^L_{11}}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x \pi^F_{11}}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} + K^L + K^F, & x \ge x^F_4. \end{cases} \tag{20}$$

In the region of $x^F_3 \le x \le x^F_4$, the value of k is consistent with Proposition 4.1. Therefore,

$$k = K^F - \frac{x(\beta-1)}{\beta}\frac{a^F(\pi^F_{00} - \pi^F_{10})}{\rho_1\rho_2}$$

As in Section 5.1, SW^{Aid} is equal to SW^C in the region of $x^L \le x \le x^F_4$. However, even if the developed country can adjust its financial aid appropriately, SW^{Aid} is not equal to SW^C in the region of $x_3^F \leq x \leq x^L$.

$$x^F(k) = \begin{cases} \underbrace{\frac{\beta}{\beta-1}\left(\frac{a\rho_1\rho_2(K^F-k^*)}{\pi_{00}^F-\pi_{10}^F}\right)}_{=x_3^F}, & x < x_3^F, \\ \frac{\beta}{\beta-1}\left(\frac{a\rho_1\rho_2(K^F-k)}{\pi_{00}^F-\pi_{10}^F}\right), & x_3^F \leq x \leq x_4^F, \\ \underbrace{\frac{\beta}{\beta-1}\left(\frac{a\rho_1\rho_2 K^F}{\pi_{00}^F-\pi_{10}^F}\right)}_{=x_4^F} & x \geq x_4^F. \end{cases}$$

Regardless of the amount of financial aid, the critical value in the developed country is given by

$$x^L = \frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2 K^L}{\pi_{00}^L-\pi_{10}^L}\right)$$

5.2.3 The noncooperative model

Social welfare in the noncooperative model is as follows:

$$SW^N(x,y) = \begin{cases} \frac{a^L x\pi_{00}^L}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x\pi_{00}^F}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} \\ \quad -\left(\frac{x}{x_4^F}\right)^\beta \left[\frac{a^L x_4^F(\pi_{10}^L-\pi_{11}^L)}{\rho_1\rho_2} + \frac{a^F x_4^F(\pi_{01}^F-\pi_{11}^F)}{\rho_1\rho_2} - K^F\right] \\ \quad -\left(\frac{x}{x^L}\right)^\beta \left[\frac{a^L x^L(\pi_{00}^L-\pi_{10}^L)}{\rho_1\rho_2} + \frac{a^F x^L(\pi_{00}^F-\pi_{01}^F)}{\rho_1\rho_2} - K^L\right], & x < x^L, \\ \frac{a^L x\pi_{10}^L}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x\pi_{01}^F}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} + K^L \\ \quad -\left(\frac{x}{x_4^F}\right)^\beta \left[\frac{a^L x_4^F(\pi_{10}^L-\pi_{11}^L)}{\rho_1\rho_2} + \frac{a^F x_4^F(\pi_{01}^F-\pi_{11}^F)}{\rho_1\rho_2} - K^F\right], & x^L \leq x \leq x_4^F, \\ \frac{a^L x\pi_{11}^L}{\rho_1\rho_2} + \frac{a^L xy}{\rho_2} + \frac{a^F x\pi_{11}^F}{\rho_1\rho_2} + \frac{a^F xy}{\rho_2} + K^L + K^F, & x \geq x_4^F, \end{cases} \tag{21}$$

where

$$x_4^F = \frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2 K^F}{a^F(\pi_{01}^F-\pi_{11}^F)}\right) = \frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2 K^F}{a^F(\pi_{00}^F-\pi_{10}^F)}\right),$$

$$x^L = \frac{\beta}{\beta-1}\left(\frac{\rho_1\rho_2 K^L}{a^L(\pi_{00}^L-\pi_{10}^L)}\right).$$

5.2.4 Comparative results and Numerical example

We obtain the following proposition by comparing the levels of social welfare implied by the three models; i.e., the cooperative model, the noncooperative model that incorporates financial aid and the noncooperative model.

Proposition 5.2. *Suppose that (AS.1)–(AS.4) hold in this scenario. Then, for $0 < x < \infty$, we obtain $SW^C \leq SW^{Aid} \leq SW^N$. $SW^C = SW^{Aid} = SW^N$ is achieved in the region of $x \geq x_4^F$. $SW^C = SW^{Aid}$ is achieved in the region of $x_4^F > x \geq x^L$.*

Proof. The results follow from equations (19),(20) and (21). □

As in Section 5.1, SW^{Aid} is equal to SW^C in the region of $x^L \leq x \leq x_4^F$. However, even if the developed country can adjust its financial aid appropriately, SW^{Aid} is not equal to SW^C in the region of $x_3^F \leq x \leq x^L$. The reason is that the timing of the developed country's policy implementation is not influenced by financial aid. Therefore, the developed country's external effect cannot be dealt with by using financial aid.

As in Subsection 5.1.4, we present numerical examples, but by using different parameter values to those used in Table 1 (see Table 3). We calculate the thresholds, x_c^L, x^L, x_3^F and

Table 3: Parameter values

Parameter		Value
π_{00}^F	Initial pollutant flow in the developing country	\$1,000,000 tons/yr
π_{10}^F	Pollutant flow in the developing country	\$600,000 tons/yr
π_{11}^F	Pollutant flow in the developing country	\$500,000 tons/yr
π_{00}^L	Initial pollutant flow in the developed country	\$500,000 tons/yr
π_{01}^L	Pollutant flow in the developed country	\$300,000 tons/yr
π_{11}^L	Pollutant flow in the developing country	\$100,000 tons/yr
π_{10}^L	Pollutant flow in the developed country	\$300,000 tons/yr
π_{01}^F	Pollutant flow in the developing country	\$900,000 tons/yr

x_4^F (see Table 4). These imply that if the current value of x_0 is \$23 per ton, the countries that do not implement the environmental policy are the developed and developing countries in the noncooperative model, the developed and developing countries in the noncooperative model that incorporates financial aid and the developing country in the noncooperative model. When x reaches \$24 per ton, the countries that implement the environmental policy are the developed and developing countries in the noncooperative model and the developed and developing countries in the noncooperative model that incorporates financial aid.

Figure 4 shows the optimal amount of financial aid with respect to the variable x. In addition, for $x \in [0, x_3^F]$, the optimal amount of financial aid is k^*; i.e., \$1.25 billion. For $x \in (x_3^F, x_4^F]$, the optimal level of financial aid is appropriately adjusted. For $x \in (x_4^F, \infty)$, the developed country need not provide financial aid to the developing country.

Figure 5 illustrates $SW^C - SW^{Aid}$. For $0 < x < x^L$, $SW^C < SW^{Aid}$ holds for two reasons. First, the developed country does not voluntarily give unlimited financial aid to the developing country. This implies that $x_c^F = x_3^F$ does not necessarily hold. Therefore, around the point of x_c^F, the difference between SW^C and SW^{Aid} increases. Second, there is a full external effect from the developed country because the critical value of the developed country is not influenced by financial aid. This implies that the developed country does not implement the environmental policy with the timing implied by the cooperative model. Therefore, around x_c^L, the gap $SW^C - SW^{Aid}$ increases. For $x_3^F < x < x_4^F$, the developed country appropriately adjusts the level of financial aid with respect to x. However, for this region, Scenario 2 differs from Scenario 1 because there is a social welfare loss. Figure 5 shows that, for $x^L < x$, $SW^C = SW^{Aid}$ holds. Thus, the level of social welfare implied by the noncooperative model that incorporates financial aid is optimal for welfare in this region. If the developed country implements the policy on x_c^L (e.g., by using an emission trading system or by setting an environmental tax for a group of developed countries), then, as in Scenario 1, for $x_3^F < x$, $SW^C = SW^{Aid}$ holds. However, social welfare in the noncooperative model that incorporates financial aid is higher than in the noncooperative model.

Figure 6 illustrates $SW^{Aid} - SW^N$. The line in the upper part of the figure denotes the benefit acquired by the developed country by providing financial aid. The gap between the two lines indicates that the benefit is shared with the developing country.

Table 4: Thresholds

x_c^F	x_3^F	x_c^L	x^L	x_4^F
12	15	20.571	24	30

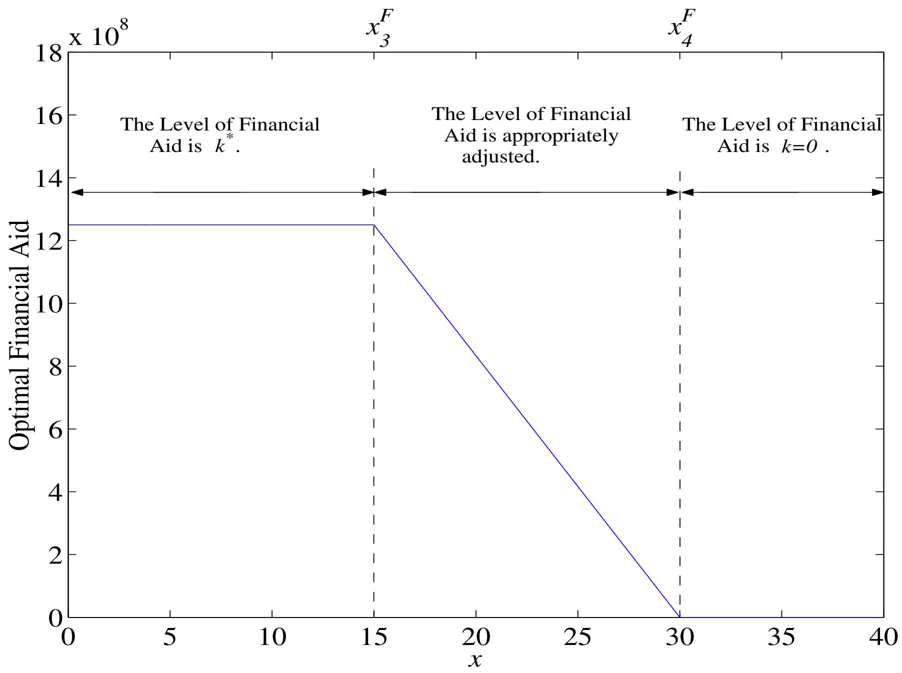

Figure 4: Optimal amount of financial aid.

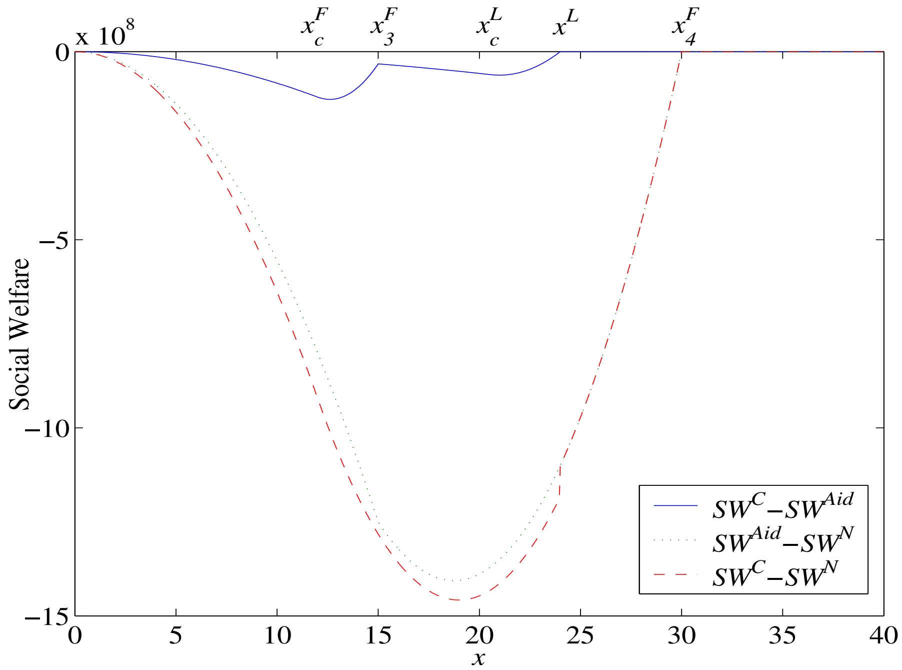

Figure 5: Comparing social welfare.

6 Conclusion

Our analysis has yielded two important findings. First, we derived the closed solution for the optimal level financial aid. This optimal level of aid depends on the stochastic variable that governs the effect of pollution stocks on the social costs of environmental damage over time. Second, the social welfare level in a noncooperative model that incorporates financial aid differs from the level implied by a cooperative model. There are two reasons for this. First, the external effect of the developed country cannot be reduced by providing financial aid. Second, the developed country does not voluntarily give unlimited financial aid to the developing country. However, social welfare is higher in a noncooperative model that incorporates financial aid than in a noncooperative model. ¿From the standpoint of the developed country, the financial-aid framework is self-motivated, rather than forced. Therefore, we suggest that financial aid is a viable solution to the problems caused by the external effects of environmental policy.

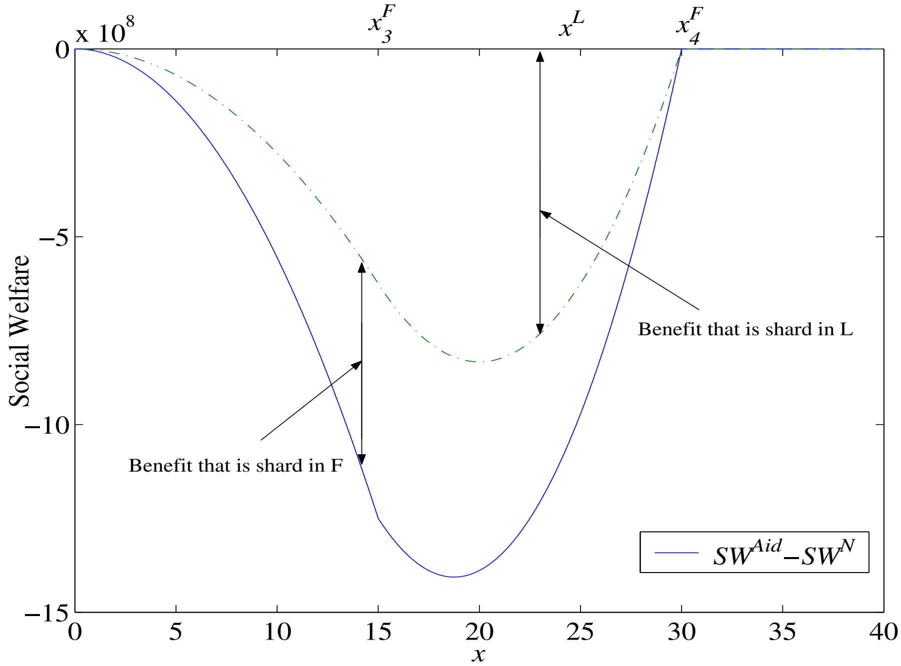

Figure 6: The difference between the social welfare levels SW^{Aid} and SW^N. (In addition, this shows that the benefit is shared between the countries.)

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