# R & D Competition in Alternative Technologies: A Real Options Approach*

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**Abstract**

We study a problem of R&D competition using a real options approach. We extend the analysis of Weeds [26] in which the project is of a fixed standard to the case where the firms can choose the target of the research from two alternative technologies of different standards. We show that the competition affects not only the firms' investment time, but also their choice of the standard of the technology. Two typical cases, namely the de facto standard case and the innovative case, are examined in full detail. In particular, in the de facto standard case, the firms could develop a lower-standard technology that would never appear in a noncompetitive situation. This provides a good account of a real problem resulting from too bitter R&D competition.

**Keywords:** Real options; Investment problem; Competition; R&D; Preemption; Optimal stopping.

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# 1 Introduction

Real options approaches have become a useful tool for evaluating irreversible investment under uncertainty such as R&D investment (see [6]). Although the early literature on real options (e.g., [4, 20]) treated the investment decision of a single firm, more recent studies have investigated the problem of several firms competing in the same market from a game theoretic approach (see [1] for an overview). Grenadier [9] derived the equilibrium investment strategies of the firms in the Cournot–Nash framework and Weeds [26] provided the asymmetric outcome (called *preemption equilibrium*) in R&D competition between the two firms using the equilibrium in a timing game studied in [8]. In [13, 25], a possibility of mistaken simultaneous investment resulting from an absence of rent equalization that was assumed in [26] was investigated.

On the other hand, there are several studies on the decision of a single firm with an option to choose both the type and the timing of the investment projects. In this literature, [5] was the first study to pay attention to the problem and Décamps et al. [3] investigated the problem in more detail. In [7], a similar model is applied to the problem of constructing small wind power units.

Despite such active studies on real options, to our knowledge few studies have tried to elucidate how competition between two firms affects their investment decisions in the case where the firms have the option to choose both the type and the timing of the projects. This paper investigates the above problem by extending the R&D model in [26] to a model where the firms can choose the target of the research from two alternative technologies of different standards with the same uncertainty about the market demand[1], where the technology standard is to be defined in some appropriate sense. As in [26], technological uncertainty is taken into account, in addition to the product market uncertainty. We assume that the time between project initiation and project discovery (henceforth the research term) follows the Poisson distribution[2] with its hazard rate determined by the standard of the technology. This assumption is realistically intuitive since a higher-standard technology is likely to require a longer research term and is

[1]We assume that the two technologies are applied to homogeneous products.

[2]Most studies, such as [2, 14, 19, 26], model technical innovation as a Poisson arrival; we also follow this convention.

expected to generate higher profits at its completion. In the model, we show that the competition between the two firms affects not only the firms' investment time, but also their choice of the technology targeted in the project.

We highlight two typical cases that are often observed in a market and, at the same time, reveal interesting implications. The first case is that a firm that completes a technology first can monopolize the profit flow regardless of the standard of the technology. *De facto standardization struggles* such as VHS vs Betamax for video recorders are true for this case (henceforth called the de facto standard case). In such cases, a firm can impose its technology as a de facto standard by introducing it before its competitors. Once one technology becomes the de facto standard for the market, the winner may well enjoy a monopolistic cash flow from the patent of the de facto standard technology for a long term. It is then quite difficult for other firms to replace it with other technologies even if those technologies are superior to the de facto standard one. Indeed, it has been often observed in de facto standardization races that the existing technology drives out a newer (superior) technology, which can be regarded as a sort of *Gresham's law*[3]. In conclusion, what is important in the de facto standard case is introducing the completed technology into the market before the opponents.

The other case is where a firm with higher-standard technology can deprive a firm with lower-standard technology of the cash flow by completing the higher-standard technology. This case applies to technologies of the innovative type (henceforth called the innovative case). As observed in evolution from cassette-based Walkmans to CD- and MD-based Walkmans, and further to flash memory- and hard drive-based digital audio players (e.g., iPod), the appearance of a newer technology drives out the existing technology. In such cases, a firm often attempts to develop a higher-standard technology because it fears the invention of superior technologies by its competitors. As a result, in the innovative case, a higher-standard technology tends to appear in a market.

The analysis in the two cases gives a good account of the characteristics mentioned above. In the de facto standard case, the competition increases the incentive to develop the lower-

[3]Gresham's law is the economic principle that in the circulation of money "bad money drives out good," i.e., when depreciated, mutilated, or debased coinage (or currency) is in concurrent circulation with money of high value in terms of precious metals, the good money is withdrawn from circulation by hoarders.

standard technology, which is easy to complete, while in the innovative case, the competition increases the incentive to develop the higher-standard technology, which is difficult to complete. In particular, we show that in the de facto standard case the competition is likely to lead the firms to invest in the lower-standard technology, which is never chosen in the single firm situation. This result explains a real problem caused by too bitter R&D competition. It is a possibility that the competition spoils the higher-standard technology that consumers would prefer[4], while the development hastened by the competition increases consumers' profits compared with that of the monopoly. That is, the result accounts for both positive and negative sides of the R&D competition for consumers. Of course, as described in [23], practical R&D management is often much more flexible and complex (e.g., growth and sequential options studied in [18, 17]) than the simple model in this paper. However, it is likely that the essence of the results remains unchanged in more practical setups.

The paper is organized as follows. After Section 2 derives the optimal investment timing for the single firm, Section 3 formulates the problem of the R&D competition between two firms. Section 4 derives the equilibrium strategies in the two typical cases, namely, the de facto standard case and the innovative case. Section 5 gives numerical examples, and finally Section 6 concludes the paper.

## 2 Single firm situation

Throughout the paper, we assume all stochastic processes and random variables are defined on the filtered probability space $(\Omega, \mathcal{F}, P; \mathcal{F}_t)$. This paper is based on the model in [26]. This section considers the investment decision of the single firm without fear of preemption. The firm can set up a research project for developing a new technology $i$ (we denote technologies 1 and 2 for the lower-standard and higher-standard technologies, respectively) by paying an indivisible investment cost $k_i$. As in [26], for analytical advantage we assume that the firm has neither option to suspend nor option to switch the projects, though practical R&D investment often allows more managerial flexibility, such as to abandon, expand and switch (see [23]).

[4] It is reasonable to suppose that consumers benefit from the invention of higher-standard technologies, though, strictly speaking, we need to incorporate consumers' value functions into the model.

In developing technology $i$, from the time of the investment the invention takes place randomly according to a Poisson distribution with constant hazard rate $h_i$. The firm must pay the research expense $l_i$ per unit of time during the research term and can receive the profit flow $D_iY(t)$ from the discovery. Here, $Y(t)$ represents a market demand of the technologies at time $t$ and influence cash flows which the technologies generate. It must be noted that the firm's R&D investment is affected by two different types of uncertainty (i.e., technological uncertainty and product market uncertainty). For simplicity, $Y(t)$ obeys the following geometric Brownian motion, which is independent of the Poisson processes representing technological uncertainty.

$$\begin{aligned} \mathrm{d}Y(t) &= \mu Y(t)\mathrm{d}t + \sigma Y(t)\mathrm{d}B(t) \quad (t > 0), \\ Y(0) &= y, \end{aligned}$$

where $\mu \geq 0, \sigma > 0$ and $y > 0$ are given constants and $B(t)$ denotes the one-dimensional $\mathcal{F}_t$ standard Brownian motion. Quantities $k_i, h_i, D_i$ and $l_i$ are given constants satisfying

$$0 \leq k_1 \leq k_2,\ 0 < h_2 < h_1,\ 0 < D_1 < D_2,\ 0 < l_1 \leq l_2, \tag{1}$$

so that technology 2 is more difficult to develop and generates a higher profit flow from its completion than technology 1.

Let us now comment upon the model. For analysis in later sections, we modified the original setup by [26] at the two following points, but there are no essential difference. In [26], the completed technology generates not a profit flow but a momentary profit as the value of the patent at its completion, and there is no research expense during the research term (i.e, $l_i = 0$). In [14] the Poisson process determining technological innovation is exogenous to the firms as in [10], but we assume that a firm's investment initiates the Poisson process determining the completion of the technology. This is the main difference from the model studied in [14] that also treats two technologies.

The firm that monitors the state of the market can set up development of either technologies 1 or 2 at the optimal timing maximizing the expected payoff under discount rate $r(> \mu)$. Then, the firm's problem is expressed as the following optimal stopping problem:

$$\sup_{\tau \in \mathcal{T}} E\left[\max_{i=1,2} E[\int_{\tau+t_i}^{\infty} \mathrm{e}^{-rt} D_i Y(t)\mathrm{d}t - \mathrm{e}^{-r\tau} k_i - \int_{\tau}^{\tau+t_i} \mathrm{e}^{-rt} l_i \mathrm{d}t \mid \mathcal{F}_\tau]\right], \tag{2}$$

where $\mathcal{T}$ is a set of all $\mathcal{F}_t$ stopping times and $t_i$ denotes a random variable representing an the Poisson arrival with hazard rate $h_i$ independent of $B(t)$. In problem (2), $\max_{i=1,2} E[\cdots \mid \mathcal{F}_\tau]$ means that the firm can choose the optimal technology at the investment time $\tau$.

By the calculation

$$
\begin{aligned}
& E[\int_{\tau+t_i}^{\infty} \mathrm{e}^{-rt} D_i Y(t)\mathrm{d}t - \mathrm{e}^{-r\tau} k_i - \int_{\tau}^{\tau+t_i} \mathrm{e}^{-rt} l_i \mathrm{d}t \mid \mathcal{F}_\tau] && (3)\\
= \;& \mathrm{e}^{-r\tau} E^{Y(\tau)}[\int_{t_i}^{\infty} \mathrm{e}^{-rt} D_i Y(t)\mathrm{d}t - k_i - \int_0^{t_i} \mathrm{e}^{-rt} l_i \mathrm{d}t] && (4)\\
= \;& \mathrm{e}^{-r\tau} \int_0^{\infty} \left( \int_s^{\infty} \mathrm{e}^{-rt} D_i E^{Y(\tau)}[Y(t)]\mathrm{d}t - k_i - \int_0^{s} \mathrm{e}^{-rt} l_i \mathrm{d}t \right) h_i \mathrm{e}^{-h_i s} \mathrm{d}s && (5)\\
= \;& \mathrm{e}^{-r\tau}(a_{i0} Y(\tau) - I_i) && (6)
\end{aligned}
$$

(we use the strong Markov property of $Y(t)$ in (4) and independence between $t_i$ and $Y(t)$ in (5) ), problem (2) can be reduced to

$$\sup_{\tau \in \mathcal{T}} E[\mathrm{e}^{-r\tau} \max_{i=1,2}(a_{i0} Y(\tau) - I_i)], \tag{7}$$

where $a_{i0}$ and $I_i$ are defined by

$$
\begin{aligned}
a_{i0} &= \frac{D_i h_i}{(r-\mu)(r+h_i-\mu)} && (8)\\
I_i &= k_i + \frac{l_i}{r+h_i}. && (9)
\end{aligned}
$$

Here, $a_{i0} Y(\tau)$ represents the expected discounted value of the future profit generated by technology $i$ at the investment time $\tau$, and $I_i$ represents its total expected discounted cost at time $\tau$. Eq. (1) and (9) imply $I_1 < I_2$, but the inequality $a_{10} < a_{20}$ does not necessarily hold depending upon a trade-off between $h_i$ and $D_i$. Note that (7) is essentially the same as the problem examined in [3]. Let $V_0(y)$ and $\tau_0^*$ denote the value function and the optimal stopping time of problem (7), respectively. Notice that $\tau_0^*$ is expressed in a form independent of the initial value $y$. As in most real options literature (e.g., [6]), we define

$$\beta_{10} = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1, \tag{10}$$

$$\beta_{20} = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0. \tag{11}$$

**Proposition 2.1** The value function $V_0(y)$ and the optimal stopping time $\tau_0^*$ of the single firm's problem (7) are given as follows:

**Case 1:** $0 < a_{20}/a_{10} \leq 1$

$$V_0(y) = \begin{cases} A_0 y^{\beta_{10}} & (0 < y < y_{10}^*) \\ a_{10}y - I_1 & (y \geq y_{10}^*), \end{cases} \tag{12}$$

$$\tau_0^* = \inf\{t \geq 0 \mid Y(t) \geq y_{10}^*\}. \tag{13}$$

**Case 2:** $1 < (a_{20}/a_{10})^{\beta_{10}/(\beta_{10}-1)} < I_2/I_1$

$$V_0(y) = \begin{cases} A_0 y^{\beta_{10}} & (0 < y < y_{10}^*) \\ a_{10}y - I_1 & (y_{10}^* \leq y \leq y_{20}^*) \\ B_0 y^{\beta_{10}} + C_0 y^{\beta_{20}} & (y_{20}^* < y < y_{30}^*) \\ a_{20}y - I_2 & (y \geq y_{30}^*), \end{cases} \tag{14}$$

$$\tau_0^* = \inf\{t \geq 0 \mid Y(t) \in [y_{10}^*, y_{20}^*] \cup [y_{30}^*, +\infty)\}. \tag{15}$$

**Case 3:** $(a_{20}/a_{10})^{\beta_{10}/(\beta_{10}-1)} \geq I_2/I_1$

$$V_0(y) = \begin{cases} B_0 y^{\beta_{10}} & (0 < y < y_{30}^*) \\ a_{20}y - I_2 & (y \geq y_{30}^*), \end{cases} \tag{16}$$

$$\tau_0^* = \inf\{t \geq 0 \mid Y(t) \geq y_{30}^*\}. \tag{17}$$

Here, constants $A_0, B_0, C_0$ and thresholds $y_{10}^*, y_{20}^*, y_{30}^*$ are determined by imposing value matching and smooth pasting conditions (see [6]). Note that $I_1 < I_2$ and $\beta_{10} > 1$.

**Proof** In Case 2, (14) and (15) immediately follows from the discussion in [3]. In Case 1, using relationships $a_{10} \geq a_{20}, I_1 < I_2$ and $Y(t) > 0$, we have

$$\sup_{\tau \in \mathcal{T}} E[\mathrm{e}^{-r\tau} \max_{i=1,2}(a_{i0}Y(\tau) - I_i)] = \sup_{\tau \in \mathcal{T}} E[\mathrm{e}^{-r\tau}(a_{10}Y(\tau) - I_1)],$$

which implies (12) and (13). In Case 3, by taking into consideration that the right-hand side of (16) dominates $a_{10}y - I_1$, we can show (16) and (17) by a standard technique to solve an optimal stopping problem (see [22]). □

In Proposition 2.1, $A_0 y^{\beta_{10}}, B_0 y^{\beta_{10}}$ and $C_0 y^{\beta_{20}}$ correspond to the values of the option to invest in technology 1 at the trigger $y^*_{10}$, the option to invest in technology 2 at the trigger $y^*_{30}$ and the option to invest in technology 1 at the trigger $y^*_{20}$, respectively. In Case 1, where the expected discounted profit of technology 1 is higher than that of technology 2, the firm initiates development of technology 1 at time (13) independently of the initial market demand $y$. In Case 3, where technology 2 is much superior to technology 1, on the contrary, the firm invests in technology 2 at time (17) regardless of $y$. In Case 2, where both projects has similar values by the trade-off between the profitability and the research term and cost, the firm's optimal investment strategy has three thresholds $y^*_{10}, y^*_{20}$ and $y^*_{30}$, and therefore the project chosen by the firm depends on the initial value $y$. Above all, if $y \in (y^*_{20}, y^*_{30})$, the firm defers not only investment, but also choice among the two projects (i.e, whether the firm invests in technology 2 when the market demand $Y(t)$ increases to the upper trigger $y^*_{30}$ or invests in technology 1 when $Y(t)$ decreases to the lower trigger $y^*_{20}$).

By letting volatility $\sigma \to +\infty$ with other parameters fixed, we have $\beta_{10} \to +1$ by definition (10) and therefore $(a_{20}/a_{10})^{\beta_{10}/(\beta_{10}-1)} \to +\infty$ if $a_{10} < a_{20}$. As a result, with high product market uncertainty, instead of Case 2, Case 3 holds and the firm chooses the higher-standard technology 2 rather than the lower-standard technology 1, unless the expected discounted profit generated by technology 1 exceeds that of technology 2. The similar result has also been mentioned in [3].

## 3 Situation of two noncooperative firms

We turn now to a problem of two symmetric firms. This paper considers a symmetric setting to avoid unnecessary confusion, but the results in this paper could remain true to some extent in an asymmetric case. For a standard discussion of an asymmetric situation, see [13]. We assume that two Poisson processes modeling the two firms' innovation are independent of each other, which means that the progress of the research project by one of the firms does not affect that of its rival. The scenarios of the cash flows into the firms can be classified into four cases. Figure 1 illustrates the cash flows into the firm that has completed a technology first (denoted Firm 1) and the other (denoted Firm 2). In the period when a single firm has succeeded in the development of technology $i$, the firm obtains the monopoly cash flow $D_i Y(t)$. If both firms develop the same

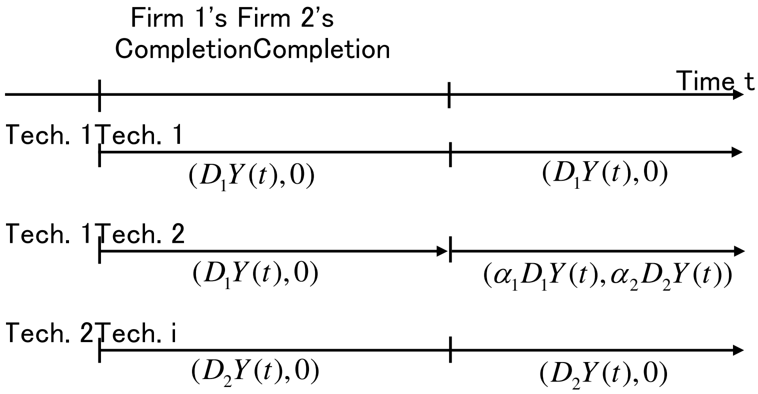


Figure 1: (Firm 1's cash flow, Firm 2's cash flow)

technology $i$, the one that has completed first receives the profit flow $D_iY(t)$ resulting from the patent perpetually and the other obtains nothing, according to the setup by [26]. Of course, the firm that has completed the lower-standard technology 1 after the competitor's completion of the higher-standard technology 2 obtains no cash flow. When the firm has completed technology 2 after the competitor's completion of technology 1, from the point technology 2 generates the profit flow $\alpha_2D_2Y(t)$, and technology 1 generates $\alpha_1D_1Y(t)$, where $\alpha_i$ is a constant satisfying $0 \le \alpha_1, \alpha_2 \le 1$. It is considered that the technology's share in the product market determines $\alpha_1$ and $\alpha_2$.

As in [6, 13, 26], we solve the game between two firms backwards. We begin by supposing that one of the firms has already invested, and find the optimal decision of the other. In the remainder of this paper, we call the one who has already invested *leader* and call the other *follower*, though we consider two symmetrical firms. Thereafter, in the next section, we look at the situation where neither firms has invested, and consider the decision of either as it contemplates whether to go first, knowing that the other will react in the way just calculated as the follower's optimal response. The main difference from [6, 13, 26] is that the follower's optimal response depends on the project chosen by the leader. Let $F_i(Y)$ and $\tau^*_{F_i}$ denote the expected discounted payoff (at time $t$) and the investment time of the follower responding optimally to the leader who has invested in technology $i$ at time $t$ satisfying $Y(t) = Y$. We denote by $L_i(Y)$ the expected discounted payoff (at time $t$) of the leader who has invested in technology $i$ at $Y(t) = Y$.

### 3.1 Case where the leader has invested in technology 2

This subsection derives $F_2(Y), \tau^*_{F_2}$ and $L_2(Y)$. Given that the leader has invested in technology 2 at $Y(t) = Y$, the follower's problem is expressed as the following optimal stopping problem with initial value $Y$.

$$\begin{aligned}&\sup_{\tau\in\mathcal{T}} E^Y \Bigg[\mathrm{e}^{-h_2\tau} \max\Bigg\{E^Y[1_{\{t_1<s_2\}}(\int_{\tau+t_1}^{\tau+s_2} \mathrm{e}^{-rs}D_1Y(s)\mathrm{d}s + \int_{\tau+s_2}^{\infty} \mathrm{e}^{-rs}\alpha_1 D_1 Y(s)\mathrm{d}s)\\&-\mathrm{e}^{-r\tau}k_1 - \int_{\tau}^{\tau+t_1} \mathrm{e}^{-rs}l_1\mathrm{d}s \mid \mathcal{F}_\tau],\\&E^Y[1_{\{t_2<s_2\}}\int_{\tau+t_2}^{+\infty} \mathrm{e}^{-rs}D_2Y(s)\mathrm{d}s \ -\mathrm{e}^{-r\tau}k_2 - \int_{\tau}^{\tau+t_2} \mathrm{e}^{-rs}l_2\mathrm{d}s \mid \mathcal{F}_\tau]\Bigg\}\Bigg],\end{aligned} \tag{18}$$

where $E^Y[\cdot]$ means the (conditional) expectation with initial value $Y$, and $t_i$ and $s_2$ represent the Poisson arrival with hazard rate $h_i$ (i.e., the research term of the follower choosing technology $i$) and the Poisson arrival with hazard rate $h_2$ (i.e., the interval between $\tau$ and the discovery time of the leader conditioned to be on the way to development of technology $i$ at $\tau$). Recall that the two Poisson processes of the follower and leader are independent of each other. What has to be noticed is that the follower's problem (18) is discounted by $\mathrm{e}^{-h_2\tau}$ differently from the single firm's problem (2). This is because the leader's completion of technology 2 deprives the follower of the future option to invest. As in the single firm's problem (2), $\max_{i=1,2} E^Y[\cdots \mid \mathcal{F}_\tau]$ means that the follower chooses the better project at the investment time $\tau$. Furthermore, $1_{\{t_i<s_2\}}$ denotes a defining function and means that the follower's payoff becomes nothing if the leader completes technology 2 first.

Via the similar calculation as (3)–(6) we can rewrite problem (18) as

$$\sup_{\tau\in\mathcal{T}} E^Y[\mathrm{e}^{-(r+h_2)\tau} \max_{i=1,2}(a_{i2}Y(\tau) - I_i)], \tag{19}$$

where $a_{ij}$ are defined by

$$a_{11} = \frac{D_1 h_1}{(r-\mu)(r+2h_1-\mu)}, \tag{20}$$

$$a_{12} = \frac{D_1 h_1}{(r+h_1+h_2-\mu)(r+h_2-\mu)}\left(1+\frac{\alpha_1 h_2}{r-\mu}\right), \tag{21}$$

$$a_{21} = \frac{D_2 h_2}{(r-\mu)(r+h_1+h_2-\mu)}\left(1+\frac{\alpha_2 h_1}{r+h_2-\mu}\right), \tag{22}$$

$$a_{22} = \frac{D_2 h_2}{(r-\mu)(r+2h_2-\mu)}. \tag{23}$$

Quantity $a_{ij}Y(\tau)$ represents the expected discounted value of the future cash flow of the firm that invests in technology $i$ at time $\tau$ when its opponent is on the way to development of technology $j$. From the expression (19), we can show the following proposition.

**Proposition 3.1** The follower's payoff $F_2(Y)$, investment time $\tau^*_{F_2}$ and the leader's payoff $L_2(Y)$ are given as follows:

**Case 1:** $0 < a_{22}/a_{12} \le 1$

$$\begin{aligned}
F_2(Y) &= \begin{cases} A_2 Y^{\beta_{12}} & (0 < Y < y^*_{12}) \\ a_{12}Y - I_1 & (Y \ge y^*_{12}), \end{cases} \\
\tau^*_{F_2} &= \inf\{s \ge t \mid Y(s) \ge y^*_{12}\}, \\
L_2(Y) &= \begin{cases} a_{20}Y - I_2 - \tilde{A}_2 Y^{\beta_{12}} & (0 < Y < y^*_{12}) \\ a_{21}Y - I_2 & (Y \ge y^*_{12}). \end{cases}
\end{aligned}$$

**Case 2:** $1 < (a_{22}/a_{12})^{\beta_{12}/(\beta_{12}-1)} < I_2/I_1$

$$\begin{aligned}
F_2(Y) &= \begin{cases} A_2 Y^{\beta_{12}} & (0 < Y < y^*_{12}) \\ a_{12}Y - I_1 & (y^*_{12} \le Y \le y^*_{22}) \\ B_2 Y^{\beta_{12}} + C_2 Y^{\beta_{22}} & (y^*_{22} < Y < y^*_{32}) \\ a_{22}Y - I_2 & (Y \ge y^*_{32}), \end{cases} \\
\tau^*_{F_2} &= \inf\{s \ge t \mid Y(s) \in [y^*_{12}, y^*_{22}] \cup [y^*_{32}, +\infty)\}, \\
L_2(Y) &= \begin{cases} a_{20}Y - I_2 - \tilde{A}_2 Y^{\beta_{12}} & (0 < Y < y^*_{12}) \\ a_{21}Y - I_2 & (y^*_{12} \le Y \le y^*_{22}) \\ a_{20}Y - I_2 - \tilde{B}_2 Y^{\beta_{12}} - \tilde{C}_2 Y^{\beta_{22}} & (y^*_{22} < Y < y^*_{32}) \\ a_{22}Y - I_2 & (Y \ge y^*_{32}). \end{cases}
\end{aligned}$$

**Case 3:** $(a_{22}/a_{12})^{\beta_{12}/(\beta_{12}-1)} \geq I_2/I_1$

$$\begin{aligned} F_2(Y) &= \begin{cases} B_2 Y^{\beta_{12}} & (0 < Y < y_{32}^*) \\ a_{22}Y - I_2 & (Y \geq y_{32}^*), \end{cases} \\ \tau_{F_2}^* &= \inf\{s \geq t \mid Y(s) \geq y_{32}^*\}, \\ L_2(Y) &= \begin{cases} a_{20}Y - I_2 - \tilde{B}_2 Y^{\beta_{12}} & (0 < Y < y_{32}^*) \\ a_{22}Y - I_2 & (Y \geq y_{32}^*). \end{cases} \end{aligned}$$

Here, $\beta_{12}$ and $\beta_{22}$ denote (10) and (11) replaced $r$ by $r + h_2$, respectively. Constants $A_2, B_2, C_2$ and thresholds $y_{12}^*, y_{22}^*, y_{32}^*$ are determined by both value matching and smooth pasting conditions, while constants $\tilde{A}_2, \tilde{B}_2$ and $\tilde{C}_2$ are determined by the value matching condition alone. Note that $I_1 < I_2$ and $\beta_{12} > 1$.

**Proof** Problem (19) coincides with problem (7) replaced $r$ and $a_{i0}$ by $r+h_2$ and $a_{i2}$, respectively. Thus, we easily obtain the follower's payoff $F_2(Y)$ and investment time $\tau_{F_2}^*$ in the same way as Proposition 2.1. We next consider the leader's payoff $L_2(Y)$. In Case 1 and 3, we readily have the same expression as that of [26] since the follower's trigger is single. In Case 2, we obtain the similar expression, though the calculation becomes more complicated because of the three triggers. □

Constants $A_2, B_2, C_2$ and thresholds $y_{12}^*, y_{22}^*, y_{32}^*$ in Proposition 3.1 correspond to constants $A_0, B_0, C_0$ and thresholds $y_{10}^*, y_{20}^*, y_{30}^*$ in Proposition 2.1, respectively. Let us explain the leader's payoff briefly. Constants $\tilde{A}_2, \tilde{B}_2$ and $\tilde{C}_2$ value the possibility that $Y$ rises above $y_{12}^*$ prior to the leader's completion, the possibility that $Y$ rises above $y_{32}^*$ prior to the leader's completion, and the possibility that $Y$ falls bellow $y_{22}^*$ prior to the leader's completion, respectively. Since these situations cause the follower's investment, the leader's payoff is reduced from the monopoly profit $a_{20}Y - I_2$ (see $Y \in (0, y_{12}^*)$ in Case 1, $Y \in (0, y_{12}^*) \cup (y_{22}^*, y_{32}^*)$ in Case 2, and $Y \in (0, y_{32}^*)$ in Case 3).

### 3.2 Case where the leader has invested in technology 1

We now consider $F_1(Y), \tau_{F_1}^*$ and $L_1(Y)$. In the previous subsection, i.e., in the case where the leader has chosen technology 2, the follower's opportunity to invest is completely lost at the leader's completion of technology 2. However, in the case where the leader has invested in technology 1, there remains the follower's option to invest in technology 2 from the leader's invention of technology 1 if the follower has not invested yet. Due to this option value, we need more complicated discussion in this subsection.

Let $f_1(Y)$ and $\tau^*_{f_1}$ be the expected discounted payoff and the optimal stopping time of the follower responding optimally to the leader who has already succeeded in development of technology 1 at $Y(t) = Y$. In other words, $f_1(Y)$ represents the remaining option value to invest in technology 2 after the leader's completion of technology 1. We need to derive $f_1(Y)$ and $\tau^*_{f_1}$ before analyzing $F_1(Y)$ and $\tau^*_{F_1}$. Given that the leader has already completed technology 1 at $Y(t) = Y$, the follower's problem becomes

$$\sup_{\tau\in\mathcal{T}} E^Y[\int_{\tau+t_2}^{\infty} \mathrm{e}^{-rt}\alpha_2 D_2 Y(t)\mathrm{d}t - \mathrm{e}^{-r\tau}k_2 - \int_{\tau}^{\tau+t_2} \mathrm{e}^{-rt} l_2 \mathrm{d}t], \tag{24}$$

which is equal to a problem of a firm that can develop only technology 2. In the same way as calculation (3)–(6), we can rewrite problem (24) as

$$\sup_{\tau\in\mathcal{T}} E^Y[\mathrm{e}^{-r\tau}(\alpha_2 a_{20} Y(t) - I_2)]. \tag{25}$$

It is easy to obtain the value function $f_1(Y)$ and the optimal stopping time $\tau^*_{f_1}$ of problem (25). If $\alpha_2 > 0$, then

$$f_1(Y) = \begin{cases} B' Y^{\beta_{10}} & (0 < Y < y') \\ \alpha_2 a_{20} Y - I_2 & (Y \geq y'), \end{cases} \tag{26}$$

$$\tau^*_{f_1} = \inf\{s \geq t \mid Y(s) \geq y'\}, \tag{27}$$

where $B'$ and $y'$ are constants determined by the value matching and smooth pasting conditions (we omit the explicit solutions to avoid cluttering). If $\alpha_2 = 0$, we have $f_1(Y) = 0$ and $\tau^*_{f_1} = +\infty$.

Assuming that the leader has begun developing technology 1 at $Y(t) = Y$, the follower's problem is expressed as follows:

$$\begin{aligned} &\sup_{\tau\in\mathcal{T}} E^Y \Bigg[\mathrm{e}^{-h_2\tau} \max\Bigg\{E^Y[1_{\{t_1<s_1\}} \int_{\tau+t_1}^{+\infty} \mathrm{e}^{-rs} D_1 Y(s)\mathrm{d}s - \mathrm{e}^{-r\tau}k_1 - \int_{\tau}^{\tau+t_1} \mathrm{e}^{-rs} l_1 \mathrm{d}s \mid \mathcal{F}_\tau], \\ &E^Y[1_{\{t_2<s_1\}} \int_{\tau+t_2}^{+\infty} \mathrm{e}^{-rs} D_2 Y(s)\mathrm{d}s + 1_{\{t_2\geq s_1\}} \int_{\tau+t_2}^{+\infty} \mathrm{e}^{-rs}\alpha_2 D_2 Y(s)\mathrm{d}s - \mathrm{e}^{-r\tau}k_2 \\ &- \int_{\tau}^{\tau+t_2} \mathrm{e}^{-rs} l_2 \mathrm{d}s \mid \mathcal{F}_\tau]\Bigg\} + 1_{\{\tau\geq s_1\}}\mathrm{e}^{-rs_1} f_1(Y(s_1))\Bigg]. \end{aligned} \tag{28}$$

Compared with the follower's problem (18) in the previous subsection, problem (28) has the additional term $E^Y[1_{\{\tau\geq s_1\}}e^{-rs_1}f_1(Y(s_1))]$. This term corresponds to the remaining option value of the inactive follower. As in (3)–(6), problem (28) can be reduced to

$$\sup_{\tau\in\mathcal{T}} E^Y[e^{-(r+h_1)\tau}\max_{i=1,2}(a_{i1}Y(\tau)-I_i)+1_{\{\tau\geq s_1\}}e^{-rs_1}f_1(Y(s_1))], \tag{29}$$

where $a_{11}$ and $a_{21}$ are defined by (20) and (22), respectively. Generally, problem (29), unlike (19), is difficult to solve analytically because of the additional term. In the next section, we overcome the difficulty by focusing on two typical cases, namely, the de fact standard case, where $(\alpha_1,\alpha_2)=(1,0)$, and the innovative case, where $(\alpha_1,\alpha_2)=(0,1)$.

# 4 Analysis in two typical cases

This section examines the firms' behaviour in the de fact standard case, where $(\alpha_1,\alpha_2)=(1,0)$, and the innovative case, where $(\alpha_1,\alpha_2)=(0,1)$. There might be a criticism that in the real world both $\alpha_1>0$ and $\alpha_2>0$ are usually hold and that the two cases seem to be too extreme. However, such a real case approximates to one of the two cases or has a middle property, depending on the relationship between $\alpha_1$ and $\alpha_2$, and therefore analysis in the two cases has a great significance of clarifying the essence of the problem. In order to exclude a situation where both firms mistakenly invest simultaneously[5], we assume that the initial value $y$ is small enough, that is,

**Assumption A**

$$\max_{i=1,2}(a_{i0}y-I_i)<0,$$

as in [26] when we discuss the preemption equilibrium. We moreover restrict our attention to the case where the firm always chooses the higher-standard technology 2 in the single firm situation, for the purpose of contrasting the competitive situation with the single firm situation. To put it more concretely, we assume

[5]We must distinguish between mistaken simultaneous investment and joint investment which is examined in Subsection 4.3. For a discussion of mistaken simultaneous investment, see [13, 24, 25].

**Assumption B**

$$\left(\frac{a_{20}}{a_{10}}\right)^{\frac{\beta_{10}}{\beta_{10}-1}} \geq \frac{I_2}{I_1},$$

so that Case 3 follows in Proposition 2.1.

In the first place, we analytically derive the follower's payoff $F_1(Y)$ and the leader's payoff $L_1(Y)$ in both de fact standard and innovative cases. Note that the results on $F_2(Y)$ and $L_2(Y)$ in Proposition 3.1 hold true by substituting $(\alpha_1, \alpha_2) = (1, 0)$ and $(\alpha_1, \alpha_2) = (0, 1)$ into (21) and (22). Then, using $L_1(Y)$ and $L_2(Y)$, we define

$$\begin{aligned} L(Y) &= \max_{i=1,2} L_i(Y), \\ F(Y) &= \begin{cases} F_1(Y) & (L_1(Y) > L_2(Y)) \\ F_2(Y) & (L_1(Y) \leq L_2(Y)). \end{cases} \end{aligned}$$

Comparing $L(Y)$ with $F(Y)$, we examine the situation where both firm try to preempt each other.

## 4.1 De facto standard case

Since $\alpha_2 = 0$ holds in this case, the follower's option value $f_1(Y)$ vanishes just like in Subsection 3.2. Thus, we can solve the follower's problem (29) in the same way as problem (19). Indeed, $F_1(Y)$ and $\tau^*_{F_1}$ agree with $F_2(Y)$ and $\tau^*_{F_2}$ replaced $a_{i2}, \beta_{i2}$ with $a_{i1}, \beta_{i1}$, respectively in Proposition 3.1, where $\beta_{11}(> 1)$ and $\beta_{21}(< 0)$ denote (10) and (11) replaced discount rate $r$ with $r + h_1$, respectively. Recall that $a_{11}$ and $a_{21}$ were defined by (20) and (22). In this case, we denote three thresholds corresponding to $y^*_{12}, y^*_{22}$ and $y^*_{32}$ in Proposition 3.1 by $y^*_{11}, y^*_{21}$ and $y^*_{31}$, respectively. Then, the leader's payoff $L_1(Y)$ coincides with $L_2(Y)$ replaced $a_{2i}, I_2, \beta_{i2}$ and $y^*_{i2}$ by $a_{1i}, I_1, \beta_{i1}$ and $y^*_{i1}$, respectively in Proposition 3.1.

Let us compare the follower's decision in the de facto standard case with the single firm's decision derived in Section 2. Using

$$\begin{aligned} \frac{a_{20}}{a_{10}} &= \frac{D_2 h_2 (r + h_1 - \mu)}{D_1 h_1 (r + h_2 - \mu)} \\ &> \frac{D_2 h_2 (r + h_1 + h_2 - \mu)}{D_1 h_1 (r + h_2 + h_2 - \mu)} = \frac{a_{22}}{a_{12}} \\ &> \frac{D_2 h_2 (r + h_1 + h_1 - \mu)}{D_1 h_1 (r + h_2 + h_1 - \mu)} = \frac{a_{21}}{a_{11}}, \end{aligned}$$

which result from $r - \mu > 0$ and $h_1 > h_2 > 0$, we have

$$\frac{a_{21}}{a_{11}} < \frac{a_{22}}{a_{12}} < \frac{a_{20}}{a_{10}}. \tag{30}$$

Eq. (30) states that the relative expected profit of technology 2 to technology 1 is smaller than that of the single firm case. Using $1 < \beta_{10} < \beta_{12} < \beta_{11}$, we also obtain

$$1 < \frac{\beta_{11}}{\beta_{11} - 1} < \frac{\beta_{12}}{\beta_{12} - 1} < \frac{\beta_{10}}{\beta_{10} - 1}. \tag{31}$$

Eq. (30) and (31) suggest a possibility that $(a_{2i}/a_{1i})^{\beta_{1i}/(\beta_{1i}-1)}$ exceeds $I_2/I_1$ and 1 even under Assumption B, and then the follower's optimal choice could be technology 1. In consequence, the presence of the leader increases the follower's incentive to choose the lower-standard technology 1, which is easy to complete, compared with in the single firm situation.

From $a_{i1} < a_{i2},\ r + h_2 < r + h_1$, problem formulations (19) and (29) (note that $f_1 = 0$ in the de facto standard case), it follows that

$$F_1(Y) < F_2(Y) \quad (Y > 0).$$

That is, from the follower's viewpoint, the case where the leader has chosen technology 2 is preferable to the case where the leader has chosen technology 1. This is due to that the leader who has invested in technology 1 is more likely to preempt the follower because of its short research term.

Finally, we take a look at the situation where neither firm has invested. Let us see that there exists a possibility that technology 1 can be developed owing to the competition even if technology 2 generates much more profit than technology 1 at its completion. Although, as has been pointed out, $(a_{2i}/a_{1i})^{\beta_{1i}/(\beta_{1i}-1)}$ could be larger than $I_2/I_1$ and 1 under Assumption B, we now consider the case where

$$\left(\frac{a_{2i}}{a_{1i}}\right)^{\frac{\beta_{1i}}{\beta_{1i}-1}} \geq \frac{I_2}{I_1} \tag{32}$$

holds, which means that a cash flow resulting from technology 2 is expected to be much greater than that of technology 1.

Since the initial value $Y(0) = y$ is small enough (Assumption A), in the single firm situation the firm invests in technology 2 (Assumption B) as soon as the market demand $Y(t)$ rises to the level $y_{30}^*$ (Figure 2). Development of technology 1 is meaningless because the firm without fear of preemption can defer the investment sufficiently. However, the firm with the fear of preemption by its rival will try to obtain the leader's payoff by investing a slight bit earlier than its rival when the leader's payoff $L(Y)$ is larger than the follower's payoff $F(Y)$. Repeating this process causes the investment trigger to fall to the point where $L(Y)$ is equal to $F(Y)$ ($y_P$ in Figure 3). At the point the firms are indifferent between the two roles, and then one of the firms invests at time $\inf\{t \geq 0 \mid Y(t) \geq y_P\}$ as leader, while the other invests at time $\tau_{F_i}^*$ (if there remains the option to invest) as follower. This phenomenon is *rent equalization* explained in [8, 26]. Thanks to the rent equalization, we can exclude the situation where both firms mistakenly invest at the same leader's trigger (see [13, 24, 25]). This asymmetric outcome where one of the firm becomes a leader and the other becomes a follower is called *preemption equilibrium*. If the fear of preemption hastens the investment time sufficiently (e.g., threshold $y_P$ becomes smaller than $\tilde{y}$ in Figure 2), then threshold $y_P$ becomes the intersection of $L_1(Y)$ and $F_1(Y)$ rather than the intersection of $L_2(Y)$ and $F_2(Y)$ (Figure 3). It suggests a possibility that in the preemption equilibrium the leader invests in technology 1. Needless to say, the leader is more likely to choose technology 1 if (32) is not satisfied. The above discussion gives a good account of the phenomenon observed frequently in de facto standard wars.

### 4.2 Innovative case

This subsection examines the innovative case, where $(\alpha_1, \alpha_2) = (0, 1)$ is satisfied. We now consider the follower's optimal response assuming that the leader has invested in technology 1 at $Y(t) = Y$. Let $\tilde{F}_1(Y)$ denote the payoff (strictly speaking, the expected discounted payoff at time $t$) of the follower who initiate developing technology 2 at time $\tau_{f_1}^*$ defined by (27). We can show that in the innovative case the follower's best response $\tau_{F_1}^*$ coincides with $\tau_{f_1}^*$ and also show $\tilde{F}_1(Y) = f_1(Y) = F_1(Y) = V_0(Y)$ as follows.

By $\alpha_2 = 1$, the payoff of the follower who invests in technology 2 at time $s(\geq t)$ is $a_{20}Y(s) - I_2$, whether the leader has completed technology 1 or not. Then we have $\tilde{F}_1(Y) = f_1(Y)$. Under Assumption B the single firm's value function $V_0(Y)$ is expressed as that of Case 3 in Proposition

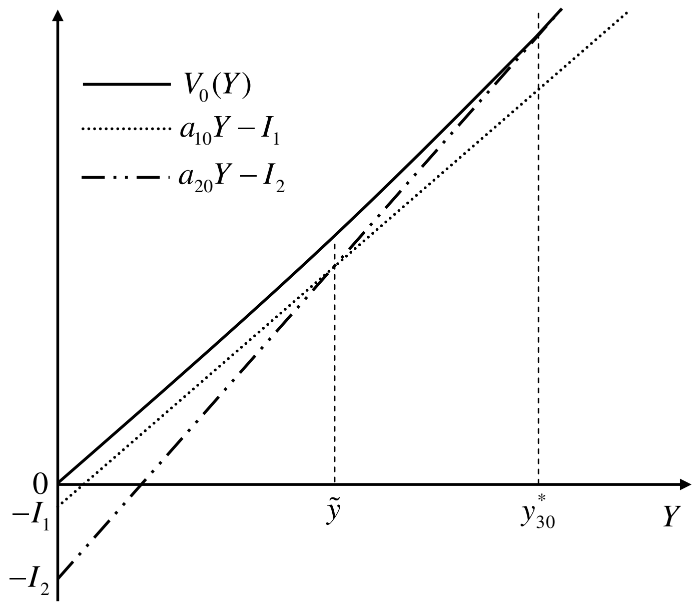


Figure 2: The value function $V_0(Y)$ in the single firm case

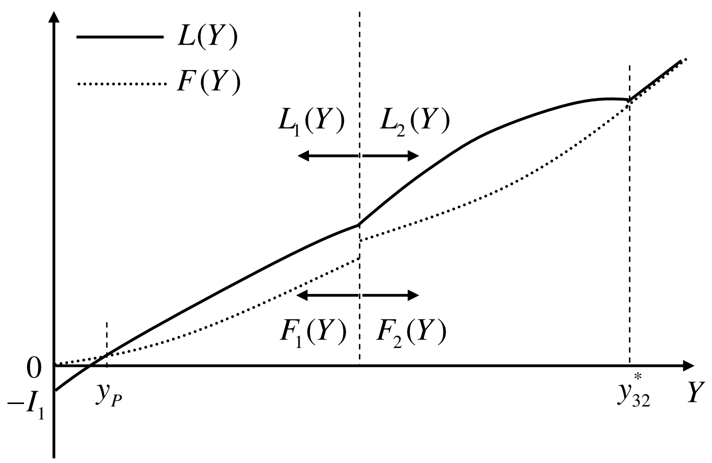


Figure 3: The leader's payoff $L(Y)$ and the follower's payoff $F(Y)$

2.1. Using $\alpha_2 = 1$, we have

$$V_0(Y) = f_1(Y) = \tilde{F}_1(Y). \tag{33}$$

On the other hand, by definition of the follower's problem (29), it can readily be seen that the relationship

$$F_1(Y) \leq V_0(Y) \tag{34}$$

holds between $F_1(Y)$ and $V_0(Y)$. Note that the follower's option value to invest in technology 2 is the same as that of the single firm case but the follower's option value to invest in technology 1 is lower than that of the single firm case owing to the fact that the follower's option value to invest in technology 1 vanishes completely at the leader's invention of technology 1. Eq. (33) and (34) suggest $F_1(Y) \leq \tilde{F}_1(Y)$. Thus, we have $\tilde{F}_1(Y) = F_1(Y)$, taking account of $F_1(Y) \geq \tilde{F}_1(Y)$ resulting from the optimality of $F_1(Y)$. Consequently, the follower's optimal response $\tau^*_{F_1}$ coincides with $\tau^*_{f_1}$ and $\tilde{F}_1(Y) = f_1(Y) = F_1(Y) = V_0(Y)$ holds. We should notice that the follower behaves as if there were no leader.

Using the follower's investment time $\tau^*_{F_2} = \tau^*_{f_1}$ derived above (note that $y' = y^*_{30}$ in (27) by $\alpha_2 = 1$), we have the leader's payoff $L_1(Y)$ as $L_2(Y)$ replaced $a_{2i}, I_2, \beta_{12}$ and $y^*_{32}$ by $a_{1i}, I_1, \beta_{11}$ and $y^*_{30}$, respectively in Case 3 in Proposition 3.1.

Next, we compare the follower's decision in the innovative case with the single firm's decision. Using

$$\begin{aligned} \frac{a_{20}}{a_{10}} &= \frac{a_{22}}{a_{12}} \times \frac{r + h_1 - \mu}{r + h_1 + h_2 - \mu} \times \frac{(r-\mu)(r + 2h_2 - \mu)}{(r + h_2 - \mu)^2} \\ &< \frac{a_{22}}{a_{12}}, \end{aligned}$$

$a_{21} = a_{20}$ and $a_{11} < a_{10}$, we have

$$1 < \frac{a_{20}}{a_{10}} < \frac{a_{2i}}{a_{1i}} \quad (i = 1, 2). \tag{35}$$

Eq. (35) means that the relative expected profit of technology 2 to technology 1 is greater than that of the single firm case, contrary to (30) in the de facto standard case. Since (31) remains true, the relationship between $(a_{22}/a_{12})^{\beta_{12}/(\beta_{12}-1)}$ and $I_2/I_1$ depends on the parameters even under Assumption B. This suggests a slight possibility that the follower chooses technology 1 in the case where the leader has chosen technology 2, while as we showed in the beginning of this subsection the follower's best response to the leader who has invested in technology 1 is choosing technology 2 regardless of $Y$. However, in most cases the effect of (35) dominates the effect of (31), that is,

$$\frac{I_2}{I_1} < \left(\frac{a_{20}}{a_{10}}\right)^{\frac{\beta_{10}}{\beta_{10}-1}} < \left(\frac{a_{22}}{a_{12}}\right)^{\frac{\beta_{12}}{\beta_{12}-1}}$$

hold. To sum up, the presence of the leader, unlike in the de facto standard case, tends to decreases the incentive of the lower-standard technology 1, which is easy to complete.

By definition of the follower's problem (19) we can immediately show

$$F_2(Y) < V_0(Y) = F_1(Y) \quad (Y > 0).$$

In other words, contrary to the de facto standard case, the follower prefers the leader developing technology 1 to the leader developing technology 2. This is because the follower can deprive the leader who has chosen technology 1 of the profit by completing technology 2.

Finally, let us examine the situation where neither firm has taken action. We obtain the following proposition with respect to the preemption equilibrium.

**Proposition 4.1** The inequality

$$L_1(Y) < F_1(Y) \quad (Y > 0) \tag{36}$$

holds, and therefore in the preemption equilibrium the leader always chooses technology 2. Furthermore, if

$$\left(\frac{a_{22}}{a_{12}}\right)^{\frac{\beta_{12}}{\beta_{12}-1}} > \frac{I_2}{I_1} \tag{37}$$

(Eq. (37) is satisfied for reasonable parameter values as mentioned earlier), then in the preemption equilibrium the follower, also, always chooses technology 2.

**Proof** The leader's payoff $L_1(Y)$ is equal to

$$L_1(Y) = \begin{cases} a_{10}Y - I_1 - \tilde{B}_1 Y^{\beta_{12}} & (0 < Y < y^*_{30}) \\ a_{12}Y - I_1 & (Y \geq y^*_{30}), \end{cases}$$

where the constant $\tilde{B}_1 > 0$ is determined by the value matching condition at the trigger $y^*_{30}$. Using $a_{10} > a_{12}$ and $\tilde{B}_1 > 0$, we have

$$L_1(Y) < a_{10}Y - I_1 \leq V_0(Y) = F_1(Y) \quad (Y > 0),$$

which implies that there is no incentive to invest in technology 1 earlier than the competitor. Therefore, there arises no preemption equilibrium where the leader invests in technology 1. Next, assume (37).

In this case, the follower's decision corresponds to that of Case 3 in Proposition 2.1. In consequence, in the preemption equilibrium, the follower, also, always chooses technology 2. □
Table 1 summarizes the comparison results between the de facto standard and innovative cases.

### 4.3 Case of joint investment

The joint investment equilibria, which are, unlike the preemption equilibria, symmetric outcomes, may also occur even if the two firms are noncooperative. The results on the joint investment equilibria in our setup become similar the results obtained in [26] and are therefore briefly described below.

Assuming that the two firms are constrained to invest in the same technology at the same timing, the firm's problem can be reduced to

$$\sup_{\tau\in\mathcal{T}} E[\mathrm{e}^{-r\tau}\max_{i=1,2}(a_{ii}Y(\tau)-I_i)], \tag{38}$$

in the same procedure as (3)–(6). Recall that $a_{11}$ and $a_{22}$ were defined by (20) and (23), respectively. It is worth noting that the expression (38) does not depend on whether the de facto standard case or the innovative case. Using

$$\begin{aligned}\frac{a_{20}}{a_{10}} &= \frac{D_2h_2(r+h_1-\mu)}{D_1h_1(r+h_2-\mu)}\\ &< \frac{D_2h_2(r+2h_1-\mu)}{D_1h_1(r+2h_2-\mu)} = \frac{a_{22}}{a_{11}}\end{aligned}$$

and Assumption B, we have

$$\frac{I_2}{I_1} < \left(\frac{a_{20}}{a_{10}}\right)^{\frac{\beta_{10}}{\beta_{10}-1}} < \left(\frac{a_{22}}{a_{11}}\right)^{\frac{\beta_{10}}{\beta_{10}-1}}.$$

Thus, the value function (denoted by $J(y)$) and the optimal stopping time (denoted by $\tau_J^*$) of problem (38) coincide with $V_0(Y)$ and $\tau_0^*$ replaced $a_{20}$ with $a_{22}$ in Case 3 in Proposition 2.1, that is, the two firms set up the development of technology 2 at the same time

$$\tau_J^* = \inf\{t \geq 0 \mid Y(t) \geq y_{33}^*\}, \tag{39}$$

where $y_{33}$ denotes the joint investment trigger corresponding to $y_{30}$ in Proposition 2.1. As in the single firm case, in joint investment both firms always choose technology 2.

If there exists any $Y$ satisfying $L(Y) > J(Y)$, then the only preemption equilibria (not necessarily unique), which are asymmetric outcomes, occur. Otherwise, there arises the joint investment equilibria (not necessarily unique) in addition to the preemption equilibria. In this case, the joint investment equilibrium attained by the optimal joint investment rule (39) Pareto-dominates the other equilibria. For further details of the joint investment equilibria, see [13, 26].

# 5 Numerical examples

This section presents some examples in which the single firm's payoff $V_0(Y)$, the leader's payoff $F(Y)$, the joint investment payoff $J(Y)$ and the equilibrium strategies are numerically computed. We set the parameter values as Table 2 in order that Assumption B is satisfied and the single firm case corresponds a standard example in [6] (note $a_{20} = I_2 = 1$). Table 3 shows $\beta_{ij}$, and Table 4 and 5 indicate $a_{ij}, I_i$ and $y^*_{ij}$. To begin with, we compute the single firm's problem. Figure 4 illustrates its value function $V_0(Y)$ corresponding to Case 3 in Proposition 2.1, where the investment time $\tau^*_0$ is

$$\tau^*_0 = \inf\{t \geq 0 \mid Y(t) \geq y^*_{30} = 2\}. \tag{40}$$

Second, let us turn to the de facto standard case. Because the inequalities

$$1 < \left(\frac{a_{2i}}{a_{1i}}\right)^{\frac{\beta_{1i}}{\beta_{1i}-1}} < \frac{I_2}{I_1} \quad (i = 1, 2)$$

hold, the follower's optimal response $\tau^*_{F_i}$ has three triggers (see Table 5), that is, which technology the follower chooses depends on the initial value $Y$. Figure 5 illustrates the leader's payoff $L_i(Y)$ and the follower's payoff $F_i(Y)$. In Figure 5, $F_i(Y)$ is smooth while $L_i(Y)$ changes drastically at the follower's triggers $y^*_{1i}, y^*_{2i}$ and $y^*_{3i}$. This means that the leader is greatly affected by the technology chosen by the follower. Particularly, a sharp rise of $L_i(Y)$ in the interval $[y^*_{2i},\ y^*_{3i}]$ in Figure 5 states that the leader prefers the follower choosing technology 2 to the follower choosing technology 1.

The payoffs $L(Y)$, $F(Y)$, and $J(Y)$ appear in Figure 6. Let us consider the firms' equilibrium strategies under Assumption A, i.e., the condition that the initial market demand $y$ is small

Table 1: Comparison between the de facto standard and innovative cases.

| | De facto standard | Innovative |
|---|---|---|
| Relative expected profit | $a_{2i}/a_{1i} < a_{20}/a_{10}$ | $a_{2i}/a_{1i} > a_{20}/a_{10}$ |
| Follower's value function | $F_1(Y) < F_2(Y)$ | $F_1(Y) > F_2(Y)$ |
| Preemption equilibrium | Both firms: likely to choose Tech. 1 | Leader: Tech. 2, Follower: Tech. 2 (in most cases) |

Table 2: Parameter setting.

| $r$ | $\mu$ | $\sigma$ | $D_1$ | $D_2$ | $h_1$ | $h_2$ | $k_1$ | $k_2$ | $l_1$ | $l_2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.04 | 0 | 0.2 | 0.025 | 0.05 | 0.32 | 0.16 | 0 | 0 | 0.18 | 0.2 |

Table 3: $\beta_{ij}$.

| $\beta_{10}$ | $\beta_{20}$ | $\beta_{11}$ | $\beta_{21}$ | $\beta_{12}$ | $\beta_{22}$ |
|---|---|---|---|---|---|
| 2 | 1 | 4.77 | −3.77 | 3.7 | −2.7 |

Table 4: Values common to both cases.

| $a_{10}$ | $a_{20}$ | $a_{11}$ | $a_{22}$ | $I_1$ | $I_2$ | $y_{30}^*$ | $y_{33}^*$ |
|---|---|---|---|---|---|---|---|
| 0.56 | 1 | 0.29 | 0.56 | 0.5 | 1 | 2 | 3.6 |

Table 5: Values dependent on the cases.

| | $a_{12}$ | $a_{21}$ | $y_{11}^*$ | $y_{21}^*$ | $y_{31}^*$ | $y_{12}^*$ | $y_{22}^*$ | $y_{32}^*$ |
|---|---|---|---|---|---|---|---|---|
| De facto standard | 0.38 | 0.38 | 2.15 | 5.46 | 5.59 | 1.78 | 2.81 | 3.04 |
| Innovative | 0.08 | 1 | N/A | N/A | 2 | N/A | N/A | 2.47 |

enough. Note that as mentioned in Section 4.3 the optimal joint investment strategy has the unique trigger $y^*_{33}$ and both firms always choose technology 2. We see from Figure 6 that the preemption equilibrium is a unique outcome in the completion between the two firms, since there exist $Y$ satisfying $J(Y) < L(Y)$. By assumption A, in the preemption equilibrium one of the firms becomes a leader investing in technology 1 at

$$\inf\{t \geq 0 \mid Y(t) \geq y_P = 0.93\} \tag{41}$$

($y_P$ is the intersection of $L(Y)$ and $F(Y)$ in Figure 6) and the other invests in technology 1 as follower at

$$\tau^*_{F_1} = \inf\{t \geq 0 \mid Y(t) \geq y^*_{11} = 2.15\}$$

if the leader has not succeeded in the development until this point. We observe that the leader's investment time (41) becomes earlier than the single firm's investment time (40). Furthermore, we see that the preemption trigger $y_P$ in Figure 6 is the intersection of $L_1(Y)$ and $F_1(Y)$ instead of that of $L_2(Y)$ and $F_2(Y)$ and see that both firms switch the target from technology 2 chosen in the single firm situation to technology 1. Thus, consumers could suffer disadvantage that the only lower-standard technology emerges due to the competition.

It is obvious from Figure 6 that in the case where the roles of the firms are exogenously given, i.e., in the leader-follower game

$$\sup_{\tau \in \mathcal{T}} E[\mathrm{e}^{-r\tau} L(Y(\tau))],$$

the leader invests in technology 1. Therefore, in this instance, rather than the fear of preemption by the competitor, the presence of the competitor causes development of the lower-standard technology 1, which is never developed in the single firm situation.

Let us now replace $\sigma = 0.2$ by $\sigma = 0.8$ with other parameters fixed in Table 2 and consider the firms' strategic behavior under Assumption A. Notice that the higher product market uncertainty $\sigma$ becomes the greater the advantage of technology 2 over technology 1 becomes. Figure 7 illustrates $L(Y), F(Y)$ and $J(Y)$. Since $J(Y) > L(Y)$ in Figure 7, the joint investment equilibria arise together with the preemption equilibria. There are two preemption equilibria corresponding the two leader's triggers $y_{P_1}$ and $y_{P_2}$. It is reasonable to suppose that which type of equilibria occurs depends on the firms' inclination to the preemption behavior. In this instance, it can be readily seen from Figure 7 that in the corresponding leader-follower game the leader invests in technology 2 at the joint investment trigger $y^*_{33}$. This suggests that relative to the case in Figure 6, the fear of preemption by the competitor could drive the leader to develop the lower-standard technology 1, which never emerges in the noncompetitive situation, at the trigger $y_{P_1}$.

Finally, we examine the innovative case. It can be deduced from the inequality

$$\left(\frac{a_{22}}{a_{12}}\right)^{\frac{\beta_{12}}{\beta_{12}-1}} > \frac{I_2}{I_1}$$

that the follower always chooses technology 1 (Table 5). The leader's payoff $L_i(Y)$ and the follower's payoff $F_i(Y)$ appear in Figure 8. The payoff $F_1(Y)$ dominates the others since it is equal to $V_0(Y)$ as shown in Section 4.2. Figure 9 illustrates $L(Y), F(Y)$ and $J(Y)$. We examine the firms' strategic behaviour under Assumption A. There occurs no joint investment outcome as there exist $Y$ satisfying $J(Y) < L(Y)$. In the preemption equilibrium, as shown in Proposition 4.1, both firms invests in the same technology 2 but the different timings. Indeed, in equilibrium one of the firms invests in technology 2 at

$$\inf\{t \geq 0 \mid Y(t) \geq y_P = 1.06\} \tag{42}$$

($y_P$ denotes the intersection of $L(Y)$ and $F(Y)$ in Figure 9) as leader, while the other invests in the same technology at

$$\tau^*_{F_2} = \inf\{t \geq 0 \mid Y(t) \geq y^*_{32} = 2.47\}$$

as follower if the leader has yet to complete the technology at this point. We see that the leader's investment time (41) is earlier than the single firm's investment time (40) but is later than (41) in the de facto standard case. The preemption trigger $y_P$ is the intersection of $L_2(Y)$ and $F_2(Y)$, and therefore the technology developed by firms remains unchanged by the competition. It is worth noting that $y_P$ agrees with the preemption trigger in the case where the firms has no option to choose technology 1, that is, the preemption trigger derived in [26].

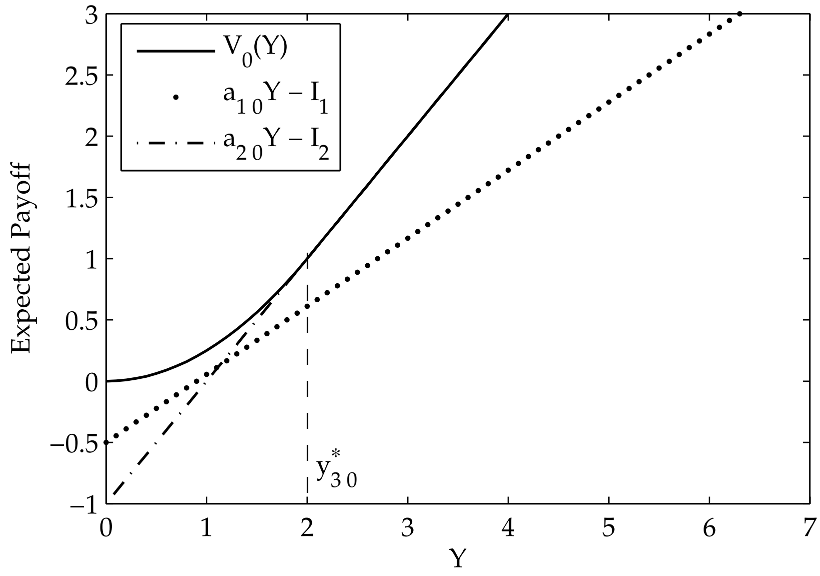


Figure 4: The single firm's value function $V_0(Y)$.

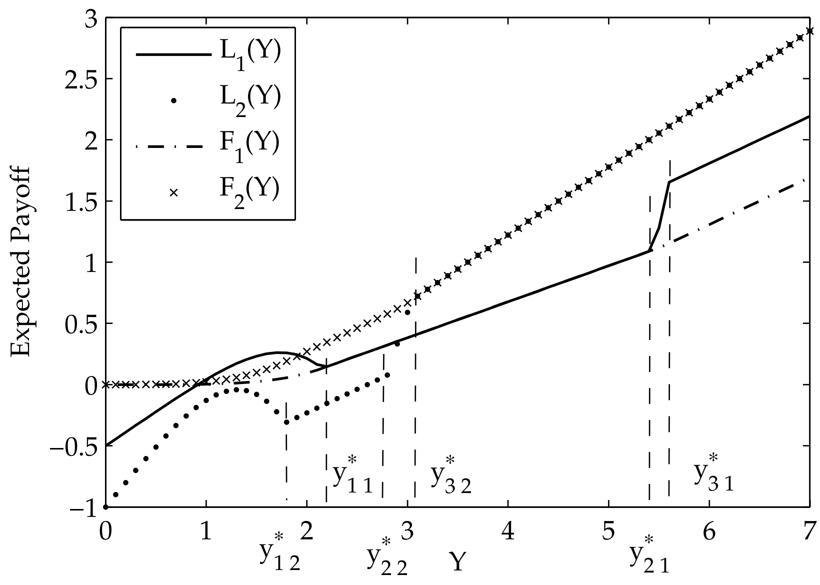


Figure 5: $L_i(Y)$ and $F_i(Y)$ in the de facto standard case.

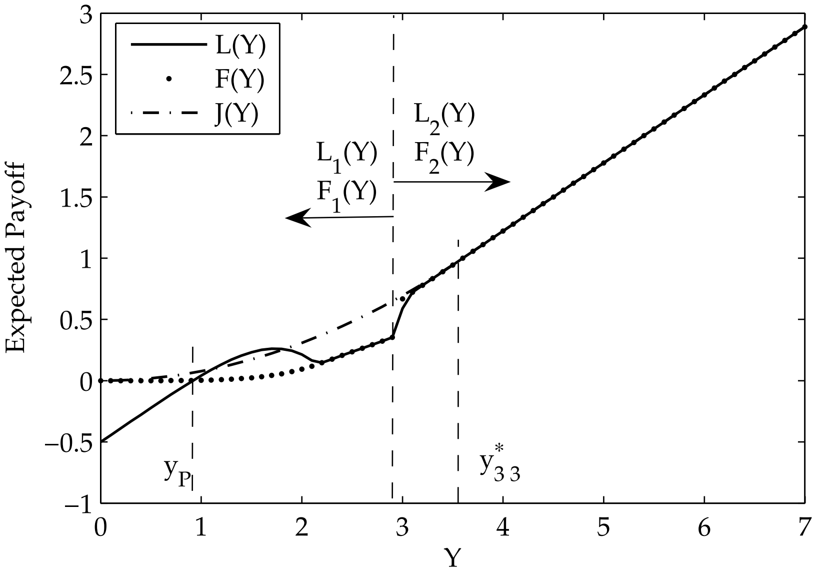


Figure 6: $L(Y), F(Y)$ and $J(Y)$ in the de facto standard case.

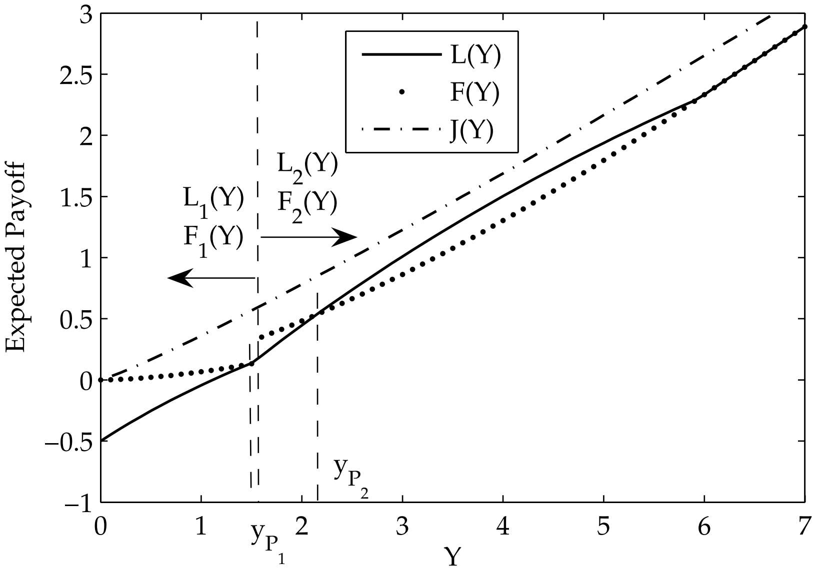


Figure 7: $L(Y), F(Y)$ and $J(Y)$ for $\sigma = 0.8$ in the de facto standard case.

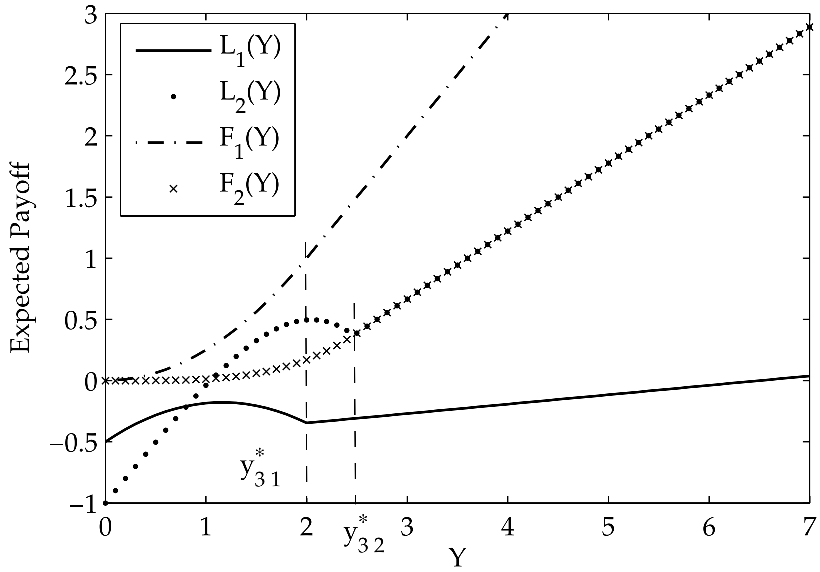


Figure 8: $L_i(Y)$ and $F_i(Y)$ in the innovative case.

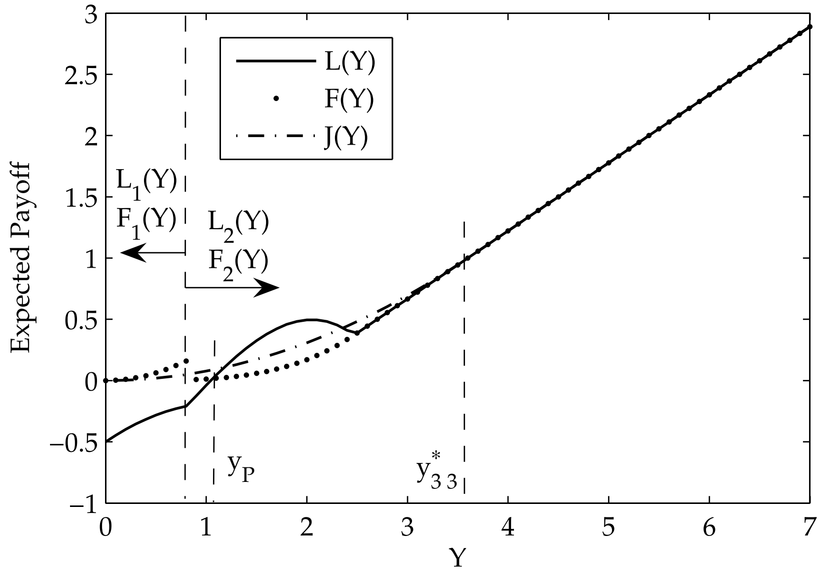


Figure 9: $L(Y), F(Y)$ and $J(Y)$ in the innovative case.

## 6 Conclusion

This paper extended the R&D model in [26] to the case where a firm has the freedom to choose the timing and the standard of the research project, where the higher-standard technology is difficult to complete and generates a greater cash flow. First, we derived the firm's optimal decision in the single firm situation. We thereafter extended the model to the situation of two firms and examined in full detail two typical cases, i.e., the de facto standard case and the innovative case. The results obtained in this paper can be summarized as follows.

In the de facto standard case, the competition increases the incentive to choose the lower-standard technology, which is easy to complete; in the innovative case, on the contrary, the competition increases the incentive to choose the higher-standard technology, which is difficult to complete. The main contribution of this paper is showing that in the de facto standard case a lower-standard technology could emerge than is developed in the single firm situation. This implies the possibility that too bitter competition among firms adversely affects not only the firms but also consumers.

Finally, we mention potential extensions of this research. One of the remaining problems is to find a system in which noncooperative firms conduct more efficient R&D investment from the viewpoint of social welfare including consumers. A tax and a subsidy investigated in [11, 15] could provide viable solutions to the problem. Although this paper considers a simple model with two types of uncertainty, namely technological uncertainty and market uncertainty, other types of uncertainty (see [12]) and other options, such as options to abandon and expand, could be involved with practical R&D investment (see [23]). It also remains as an interesting issue for future research to incorporate incomplete information (for example, uncertainty as to rivals' behavior as investigated in [16, 21]) in the model.

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