The Effects of Auditing in a Real Options Model

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Abstract

This paper investigates the owner’s optimal contract with a bonus-incentive and audit when the owner delegates the investment timing decision to a manager with private information on an investment project. The solution includes discussion of the findings obtained in previous work in this area.

Keywords: Real Options; Asymmetric Information; Agency Conflicts; Auditing.

1 Introduction

The real options approach has become an increasingly standard framework for investment timing decisions in corporate finance (see, for example, [6]). Although the early literature on real options (e.g., [5, 10]) considered the investment decisions of monopolists, more recent studies have investigated the problem of several firms competing in the same market from a game theoretic approach (see [4] for an overview). Grenadier [7] derived the equilibrium investment strategies of firms in a Cournot–Nash framework and Weeds [12] incorporated equilibrium in a timing game in a real options model.

While these studies have focused on the strategic interaction with rival firms, Grenadier and Wang [8] investigated investment timing in a decentralized firm where the owner (principal) delegates the investment decision to the manager (agent) who holds private information by

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combining the real options approach and contract theory (for contract theory, see the standard textbook [3]). In their model, asymmetric information changes the investment behavior of the firm from the first-best no-agency case because the owner designs the contract to provide a bonus-incentive for the manager to truthfully reveal private information. Similar real options models with agency conflicts have been also studied in [2, 9].

Although these models consider only the carrot (i.e., giving a bonus-incentive to the manager) as a measure to deal with agency conflicts, the owner can usually use not only the carrot, but also the stick (i.e., auditing and fining the manager). Naturally, the impact of auditing has been clarified in other contexts (e.g., [1]). In this paper, we incorporate an auditing mechanism into a model following [8]. As far as the purpose of the paper is concerned, we limit their original setup involving both hidden information and action to only the case of hidden information. We assume that the owner can utilize an auditing system that fines the manager when a false report is detected, where the higher the cost the owner pays in auditing, the greater becomes the probability of detection. We show that the optimal contract is determined among three feasible types of contracts: the bonus-incentive only mechanism, the joint bonus-incentive and auditing mechanism, and the auditing only mechanism. This is according to the relation between the auditing cost and the amount of the penalty. Although a similar auditing technology is introduced in [11], the findings in this paper are more comprehensive and help connect previous results in this area. Indeed, our solution includes the results of [8] in the bonus-incentive only region, the results of [11] in the joint bonus-incentive and auditing region, and the first-best no-agency solution as the limit of the auditing only region.

The paper is organized as follows. Section 2 provides a brief review of [8]. Section 3 incorporates the auditing technology into the model and derives the owner’s optimal contract after allowing for audit.

2 Preliminaries

This section provides a brief review of the results in [8]. First, let us explain the setup. We consider a decentralized firm that faces the investment timing decision of a single project. We assume that the owner (principal) delegates decisions to the manager (agent) and that both
the owner and the manager are risk-neutral. While both the owner and the manager know the investment cost $K$, the value of the project consists of two components, namely the value $P(t)$ that is observable to both the owner and the manager, and the value $\theta$ that is privately observed only by the manager. Thus, the total value of the project is $P(t) + \theta$. For simplicity, we assume that the observable value $P(t)$ at time $t$ obeys the following geometric Brownian motion:

$$dP(t) = \alpha P(t)dt + \sigma P(t)dz(t) \quad (t > 0), \quad P(0) = P_0,$$

where $\alpha \geq 0, \sigma > 0$ and $P_0$ are given constants, and $z(t)$ denotes the one-dimensional standard Brownian motion.

The private component $\theta$ potentially takes on two possible values, $\theta_1$ or $\theta_2$, with $0 \leq \theta_2 < \theta_1 < K$. We denote $\Delta \theta = \theta_1 - \theta_2$. Before contracting, both the owner and the manager know that the probability of drawing a higher quality project $\theta_1$ equals $q$. Immediately after making a contract with the owner at time 0, the manager privately observes whether the project is of a higher quality $\theta_1$ or a lower quality $\theta_2$. Although the manager’s one-time effort, which cannot be observed by the owner, changes the likelihood $q$ in [8], we exclude the effect of the hidden action from the original setup.

As a benchmark, we examine the case where there is no delegation of the exercise decision and the owner observes the true value of $\theta$. In this case, the owner’s problem with given $\theta = \theta_i$ becomes the following optimal stopping problem:

$$W(P_0; \theta_i) = \sup_{\tau_i \in \mathcal{T}} E[e^{-r\tau_i}(P(t) + \theta_i - K)],$$

where $\mathcal{T}$ is a set of all $\mathcal{F}_t$ stopping times, ($\mathcal{F}_t$ is the usual filtration generated by $z(t)$), and $r$ is a constant risk-free rate satisfying $r > \alpha$. In this paper, it is always assumed that the initial value $P_0$ is sufficiently low so that the firm has to wait for its exercise condition to be met. Using the standard method (see [6]), we obtain the value function $W(P_0; \theta_i)$ and the optimal stopping time $\tau^*_i$ of problem (2) for $\theta = \theta_i$ as follows:

$$W(P_0; \theta_i) = \left(\frac{P_0}{P^*_i}\right)^\beta (P^*_i + \theta_i - K)$$

$$\tau^*_i = \inf\{t \geq 0 \mid P(t) \geq P^*_i\}$$

$$P^*_i = \frac{\beta}{\beta - 1} (K - \theta_i),$$

$$3$$
where $\beta$ is defined by $
abla = \frac{1}{2} - \mu/\sigma^2 + \sqrt{(\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2} (> 1)$. The threshold $P^*_t$ is the optimal investment trigger for the owner who observes the value $\theta_i$ at time 0. Thus, the ex ante value of the owner’s option in the first-best no-agency setting (denoted $V^*(P_0)$) becomes:

$$V^*(P_0) = qW(P_0; \theta_1) + (1-q)W(P_0; \theta_2) = q \left( \frac{P_0}{P^*_t} \right)^\beta (P^*_t + \theta_1 - K) + (1-q) \left( \frac{P_0}{P^*_2} \right)^\beta (P^*_2 + \theta_2 - K).$$

(4)

Now, let us turn to the principal-agent setting without auditing. In this case, the owner has the option to invest, but delegates the exercise decision to the manager. At time 0, the owner offers the manager a contract that commits the owner to pay the manager at the time of exercise. We assume no opportunity for renegotiation exists. Although the commitment may lead to ex post inefficiency in investment timing, it increases the ex ante value of the project. In fact, if the owner makes no contract with the manager, the owner’s ex ante option value becomes (4) with $q = 0$. This is because the manager hands the owner $\theta_2$ and makes $\theta_1 - \theta_2$ his/her own when the true value is $\theta_1$. As discussed in [8], the optimal contract is included in a mechanism $\mathcal{M}^G = \{(P_i, w_i) \mid i = 1, 2\}$ in which the owner pays the manager the bonus $w_i$ at the investment time when the manager exercises the investment at time $\tau_i = \inf\{t \geq 0 \mid P(t) \geq P_i\}$. Since the revelation principal (see [3]) ensures that the manager who observes $\theta_i$ faithfully invests at the trigger $P_i$, the optimal contract is the solution of the problem of maximizing the owner’s ex ante option value of the investment:

$$\max_{P_i, w_i} \quad q \left( \frac{P_0}{P^*_1} \right)^\beta (P^*_1 + \theta_1 - K - w_1) + (1-q) \left( \frac{P_0}{P^*_2} \right)^\beta (P^*_2 + \theta_2 - K - w_2)$$

subject to

$$\begin{align*}
q \left( \frac{P_0}{P^*_1} \right)^\beta w_1 + (1-q) \left( \frac{P_0}{P^*_2} \right)^\beta w_2 & \geq 0 \\
\frac{P_0}{P^*_1} w_1 - \frac{P_0}{P^*_2} (w_2 + \Delta \theta) & \geq 0 \\
\frac{P_0}{P^*_2} w_2 - \frac{P_0}{P^*_1} (w_1 - \Delta \theta) & \geq 0,
\end{align*}$$

(5)

where $P_i > P_0$. In the constraints of problem (5), the first and second inequalities correspond to the ex ante participation constraint and the ex post limited-liability constraints, respectively, while the last two inequalities are the ex post incentive-compatibility constraints. The incentive-compatibility constraint means that with a truthful report, the manager who observes $\theta = \theta_1$
(resp. \( \theta = \theta_2 \)) obtains the expected payoff \( \left( P_0/P_1 \right)^{\beta} w_1 \) (resp. \( (P_0/P_2)^{\beta} w_2 \)), which is larger than the expected payoff for a false report, \( (P_0/P_2)^{\beta}(w_2 + \Delta \theta) \) (resp. \( (P_0/P_1)^{\beta}(w_1 - \Delta \theta) \)).

In problem (5), it can be shown that the bonus \( w_2 = 0 \) and the only third inequality (i.e., the incentive-compatibility condition for the manager who observes the better project value \( \theta_1 \)) binds. Then, the optimal solution \( \{(P_i^{GW}, w_i^{GW}) \mid i = 1, 2 \} \) becomes:

\[
(P_1^{GW}, w_1^{GW}) = \left( P_1^*, \left( \frac{P_1^*}{P_2^{GW}} \right)^{\beta} \Delta \theta \right)
\]

\[
(P_2^{GW}, w_2^{GW}) = \left( \frac{\beta}{\beta - 1} \left( K - \theta_2 + \frac{q\Delta \theta}{1-q} \right), 0 \right).
\]

For further details, see the solution for the hidden information only region in [8]. It is worth noting that the trigger for the higher quality project, \( P_1^{GW} \), remains unchanged from the first-best trigger \( P_1^* \) defined by (3), while the trigger for the lower quality project, \( P_2^{GW} \), is larger than the first-best trigger \( P_2^* \) defined by (3). This is because the owner attempts to decrease the information rent to the manager who observes the higher quality project by deferring investment timing for the lower quality project. The owner’s and manager’s ex ante option values, \( \pi_0^{GW} \) and \( \pi_m^{GW} \), respectively, are obtained by substituting the optimal solution \( \{(P_i^{GW}, w_i^{GW}) \mid i = 1, 2 \} \) into the objective function and the right-hand side of the first inequality of the constraints of problem (5).

This is the essence of the results obtained by [8]. In the next section, we extend their analysis to a case allowing the owner to audit the manager at a cost.

### 3 Main Results

This section derives the optimal contract involving the bonus-incentive and the audit. The owner detects the real value of \( \theta \) at probability \( p_i \) by paying the auditing cost \( c(p_i) \) for the manager’s report \( \theta = \theta_i \) when the manager executes the project. We assume that the manager is fined the penalty \( \Gamma(>0) \) for cheating when a false report is detected. We do not care whether or not the penalty \( \Gamma \) goes to the manager, since the manager does not in any case cheat the owner in the optimal contract. Here, the cost function \( c(p_i) \) and \( \Gamma \) are given exogenously. We assume that the cost function \( c(p_i) \) satisfies conditions \( c(0) = 0, \lim_{p_i \uparrow 1} c(p_i) = +\infty, c'(p_i) > 0 \) (\( p_i \in [0,1] \)),

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and $c''(p_i) > 0$ ($p_i \in [0, 1]$). The first and third conditions are explicit from the property of auditing. The second assumption is realistically intuitive because no auditing can always detect the manager’s false report. The final condition ensures the convexity of the cost function.

In this setting, the contract is designed as a mechanism $M^A = \{(P_i, w_i, p_i) \mid i = 1, 2\}$, where the auditing level $p_i$ for the manager’s report $\theta = \theta_i$ is added to the mechanism $M^{GW}$. Then, the optimal contract to the owner becomes the solution of the following problem:

$$
\max_{P_i, w_i, p_i} \quad q \left( \frac{P_0}{P_1} \right)^\beta (P_1 + \theta_1 - K - w_1 - c(p_1)) + (1 - q) \left( \frac{P_0}{P_2} \right)^\beta (P_2 + \theta_2 - K - w_2 - c(p_2))
$$
subject to

$$
q \left( \frac{P_0}{P_1} \right)^\beta w_1 + (1 - q) \left( \frac{P_0}{P_2} \right)^\beta w_2 \geq 0
$$
$$
w_i \geq 0 \quad (i = 1, 2)
$$
$$
p_i \geq 0 \quad (i = 1, 2)
$$

$$
\left( \frac{P_0}{P_1} \right)^\beta w_1 - \left( \frac{P_0}{P_2} \right)^\beta (w_2 + \Delta \theta - p_2 \Gamma) \geq 0
$$
$$
\left( \frac{P_0}{P_2} \right)^\beta w_2 - \left( \frac{P_0}{P_1} \right)^\beta (w_1 - \Delta \theta - p_1 \Gamma) \geq 0,
$$

where $P_i > P_0$. The first two constraints of problem (8) are the same as those of problem (5), while the incentive-compatibility constraints of problem (8) include an additional term, the expected penalty $p_i \Gamma$. It can be easily checked that the revelation principal holds in this case, as in the previous setting, even if the penalty $\Gamma$ is counted in the owner’s profit. Therefore, the owner does not make any contract to allow the manager to report untruthfully.

**Proposition 3.1** The optimal contract $\{(P_i^A, w_i^A, p_i^A) \mid (i = 1, 2)\}$ in the setting with auditing is given as follows:

**Case A:** $0 < \Gamma \leq (1 - q)c'(0)/q$ (bonus-incentive only region)

$$
(P_1^A, w_1^A, p_1^A) = (P_1^*, w_1^{GW}, 0)
$$
$$
(P_2^A, w_2^A, p_2^A) = (P_2^{GW}, 0, 0),
$$

**Case B:** $(1 - q)c'(0)/q < \Gamma \leq \max\{\Delta \theta, (1 - q)c'(\Delta \theta/\Gamma)/q\}$ (joint bonus-incentive and auditing
region)

\[
(P_1^\lambda, w_1^\lambda, p_1^\lambda) = \left( P_1^*, \left( \frac{P_1^*}{P_2^*} \right)^\beta (\Delta \theta - p_2^\lambda \Gamma), 0 \right)
\]

\[
(P_2^\lambda, w_2^\lambda, p_2^\lambda) = \left( \frac{\beta}{\beta - 1} \left( K - \theta_2 + \frac{q}{1-q} (\Delta \theta - p_2^\lambda \Gamma) + c(p_2^\lambda) \right), 0, (c')^{-1} \left( \frac{q \Gamma}{1-q} \right) \right).
\]

**Case C**: \( \Gamma > \max \{ \Delta \theta, (1-q)c'(\Delta \theta/\Gamma)/q \} \) (auditing only region)

\[
(P_1^\lambda, w_1^\lambda, p_1^\lambda) = (P_1^*, 0, 0)
\]

\[
(P_2^\lambda, w_2^\lambda, p_2^\lambda) = \left( \frac{\beta}{\beta - 1} (K - \theta_2 + c(p_2^\lambda)), 0, \Delta \theta/\Gamma \right).
\]

**Proof** Note that in problem (8), the first constraint is induced by the second constraints \( w_i \geq 0 \) \((i = 1, 2)\), and \( P_0^\beta \) can be ignored. We solve the problem (8) without the final constraint (the incentive-compatibility constraint for the manager who observes the bad value \( \theta = \theta_2 \)), and then also check that the obtained solution satisfies the removed constraint. Let \( \{ (P_i^A, w_i^A, p_i^A) \mid (i = 1, 2) \} \) be the optimal solution of problem (8) without the final constraint. It immediately follows that \( w_2^A = 0, p_1^A = 0 \), and:

\[
P_1^{A-\beta} w_1^A - P_2^{A-\beta} (\Delta \theta - p_2^A \Gamma) = 0. \tag{9}
\]

Let \( \lambda_i \) \((i = 1, 2, 3)\) denote the Lagrangian multipliers associated with the remaining constraints \( w_1 \geq 0, p_2 \geq 0 \), and (9), respectively. That is, we form the Lagrangian:

\[
\mathcal{L}(P_1, P_2, w_1, p_2) = q P_1^{-\beta} (P_1 + \theta_1 - K - w_1) + (1-q) P_2^{-\beta} (P_2 + \theta_2 - K - c(p_2))
\]

\[
+ \lambda_1 w_1 + \lambda_2 p_2 + \lambda_3 \left( P_1^{\beta-1} w_1 - P_2^{\beta-1} (\Delta \theta - p_2 \Gamma) \right).
\]

Then, in addition to (9), we have the KKT conditions

\[
\frac{\partial \mathcal{L}}{\partial P_1} = q \left( (-\beta + 1) P_1^{A-\beta} - \beta (\theta_1 - K - w_1^A) P_1^{A-\beta-1} \right) - \lambda_3 \beta w_1^A P_1^{A-\beta-1} = 0 \tag{10}
\]

\[
\frac{\partial \mathcal{L}}{\partial P_2} = (1-q) \left( (-\beta + 1) P_2^{A-\beta} - \beta (\theta_2 - K - c(p_2^A)) P_2^{A-\beta-1} \right) + \lambda_3 (\Delta \theta - p_2^A \Gamma) P_2^{A-\beta-1} \tag{11}
\]

\[
\frac{\partial \mathcal{L}}{\partial w_1} = -q P_1^{A-\beta} + \lambda_1 + \lambda_3 P_1^{A-\beta} = 0 \tag{12}
\]

\[
\frac{\partial \mathcal{L}}{\partial p_2} = -(1-q) c'(p_2^A) P_2^{A-\beta} + \lambda_2 + \lambda_3 \Gamma P_2^{A-\beta} = 0 \tag{13}
\]
and
\[
\lambda_1 w_1^A = \lambda_2 w_2^A = 0, \quad \lambda_i \geq 0 \ (i = 1, 2, 3).
\] (14)

Let us now derive the solution of (9)–(14), depending on whether \(\lambda_i\) equals zero. If \(\lambda_1 = \lambda_2 = 0\), we have \(\lambda_3 = q\) and the solution in Case B. If \(\lambda_1 > 0\) and \(\lambda_2 = 0\), from (9)–(13) we have the solution in Case C with \(\lambda_1 = P_1^A \beta (q - (1 - q)c' (p_2^A)/\Gamma)\), \(\lambda_3 = (1 - q)c' (p_2^A)/\Gamma\). If \(\lambda_1 = 0\) and \(\lambda_2 > 0\), we obtain the solution in Case A with \(\lambda_2 = P_2^A \beta ((1 - q)c' (0) - q\Gamma), \lambda_3 = q\). If \(\lambda_i > 0 \ (i = 1, 2)\), from (9) we have \(\Delta \theta = 0\), which contradicts \(\Delta \theta > 0\). Taking into account that the conditions \(\lambda_i \geq 0 \ (i = 1, 2, 3)\), \(P_2^A < 1\) that results from \(\lim_{p_2 \to 1} c(p_2) = +\infty\), and the condition under which the solution of \(c' (p_2^A) = q\Gamma/(1 - q)\) exists, we can show that for a given \(\Gamma\), the solution satisfying the KKT conditions (9)–(14) is uniquely determined as the statement of Proposition 3.1. Furthermore, the solution explicitly satisfies the final constraint in problem (8).

In Proposition 3.1, and as intuitively expected, the owner neither gives a bonus for the manager’s bad report \(\theta_2\) nor audits the manager’s good report \(\theta_1\). The investment trigger of the high quality project does not change from that of the no-agency setting as in the result by [8]. However, the other components of the contract and the owner’s strategy changes, depending on the auditing cost \(c(p)\) and the amount of the penalty, \(\Gamma\). Indeed, the contract is classified into three regions. The solution changes from Case A to Case C via Case B as the penalty \(\Gamma\) becomes larger, as observed in Figures 1 and 2. In the numerical example, we took the parameter values \(\alpha = 0, r = 0.04, \sigma = 0.2\) and \(K = 1\) as in [6], and took \(P_0 = 0.5, \theta_1 = 0.5, \theta_2 = 0, q = 0.5\) and \(c(p_i) = 0.5p_i/(1 - p_i)\).

Case A is likely to hold if the marginal cost of auditing is high relative to the penalty \(\Gamma\), or if the probability of drawing the better project \(\theta_1\) is low. In this case, the owner pays the whole information rent to the manager without auditing, since the auditing technology does not work at all. Conversely, in Case C, where the penalty is severe and the auditing cost is not so high, the owner uses only the auditing system without giving any bonus to the manager. Case B is the middle case. In this case, both the bonus-incentive and auditing are effective by the trade-off between the auditing cost and the amount of the penalty.
Let us explain the relation between Proposition 3.1 and the results from previous studies. The solution in Case A coincides with that of [8], i.e., (6) and (7). Then, it is readily seen that
\[ P^*_2 < P^A_2 \leq P^{GW}_2, \quad \pi^{GW}_o(P_0) \leq \pi^A_o(P_0) < V^*(P_0), \] and
\[ \pi^A_m(P_0) \leq \pi^{GW}_m(P_0), \] where \( \pi^A_o \) and \( \pi^A_m \) denote the owner’s and manager’s ex ante option values defined by
\[
\begin{align*}
\pi^A_o(P_0) &= q \left( \frac{P_0}{P^*_1} \right) ^\beta (P^*_1 + \theta_1 - w^A_1) + (1 - q) \left( \frac{P_0}{P^A_2} \right) ^\beta (P^A_2 + \theta_2 - K - c(P^A_2)) \\
\pi^A_m(P_0) &= q \left( \frac{P^*_1}{P^A_2} \right) ^\beta w^A_1.
\end{align*}
\]
Note that \( \pi^A_o \) and \( \pi^A_m \) are exactly the same as \( \pi^{GW}_o \) and \( \pi^{GW}_m \), respectively, in Case A. If we assume that \( \Gamma = \Delta \theta \) and \( c'(0) = 0 \), the solution is classified into Case B and agrees with that of [11] where the owner can control the penalty \( \Gamma \) to a limit \( \Delta \theta \). Moreover, the solution in Case C converges to that of the first-best no-agency case by letting \( \Gamma \uparrow +\infty \). This appears to correspond to Proposition 4 in [1].

We can see from Figure 1 that \( P^A_2 \) monotonically decreases to \( P^*_2 = 2 \) for penalties \( \Gamma \). Figure 2 indicates that the owner’s (resp. manager’s) ex ante option value \( \pi^A_o \) (resp. \( \pi^A_m \)) monotonically increases (resp. decreases) for \( \Gamma \). These results can also be easily proved, and therefore the proofs are omitted. In particular, the monotone increase in the owner’s option with respect to the amount of the penalty is consistent with the maximal punishment principal (see [1, 11]). That is, the owner imposes the maximum penalty if he/she can determine the penalty within some limit. On the other hand, the social loss \( L^A \) defined by \( L^A = V^* - (\pi^A_o + \pi^A_m) \) as in [8], is not necessarily monotonic for \( \Gamma \) as observed in Figure 2, although for sufficiently large \( \Gamma \), it decreases monotonically to zero, i.e., \( L^A \downarrow 0 \) as \( \Gamma \uparrow +\infty \) (cf. Proposition 4 in [1]). This means that a halfway penalty could only be of benefit to the owner and brings with it inefficiency in terms of social surplus, while a severe penalty improves social efficiency. Although this paper focuses on the characterization of the optimal solution, we intend to examine the economic implications and applications of the model as future research directions.

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Figure 1: $P^A_2$, $w_1^A$ and $c(p^A_2)$ for penalties $\Gamma$.

Figure 2: $\pi^A_o(P_0)$, $\pi^A_m(P_0)$ and $L^A(P_0)$ for penalties $\Gamma$. 
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References


