物理統計学

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$V(t)$ はオルンシュタイン・ウーレンベック過程で，次のランジュバン方程式に従う．

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t} = -\gamma V(t) + f(t)$$

ここで，γ は正の定数．$f(t)$ は白色雑音で，$\langle f(t)\rangle = 0$，$\langle f(t)f(s)\rangle = 2D\delta(t-s)$．（$\langle A\rangle$ は A の平均を表し，D は正の定数，$\delta(t)$ はディラックのデルタ関数.）

(i) $V(t_0) = V_0$（V_0 は定数）のとき $V(t) = V_0 e^{-\gamma(t-t_0)} + \int_{t_0}^{t} \mathrm{d}s e^{\gamma(s-t)} f(s)$ を示せ.

(ii) $\langle V(t)\rangle$ と $\sigma(t)^2 = \langle V(t)^2\rangle - \langle V(t)\rangle^2$ を求めよ.

(iii) 定常状態での時間相関関数 $\varphi_V(t) = \lim_{s\to\infty} \langle V(t+s)V(s)\rangle$ を求めよ.

(vi) $V(t)$ の確率密度関数 $P(v,t)$ に関するフォッカー・プランク方程式を導き，定常解 $P_{\mathrm{st}}(v)$ を求めよ.

Physical Statistics

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Let $V(t)$ be the Ornstein-Uhlenbeck process which obeys the following Langevin equation,

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t} = -\gamma V(t) + f(t),$$

where γ is a positive constant and $f(t)$ the white noise, whose mean and correlation function are given by $\langle f(t)\rangle = 0$ and $\langle f(t)f(s)\rangle = 2D\delta(t-s)$, respectively. Here, $\langle A\rangle$ denotes the average of A, D a positive constant and $\delta(t)$ Dirac's delta function.

(i) Show that $V(t) = V_0 e^{-\gamma(t-t_0)} + \int_{t_0}^{t} \mathrm{d}s e^{\gamma(s-t)} f(s)$ for $V(t_0) = V_0$, where V_0 is a constant.

(ii) Evaluate $\langle V(t)\rangle$ and $\sigma(t)^2 = \langle V(t)^2\rangle - \langle V(t)\rangle^2$.

(iii) Find the time-correlation function at the stationary state, $\varphi_V(t) = \lim_{s\to\infty} \langle V(t+s)V(s)\rangle$.

(iv) Derive the Fokker-Planck equation for the probability density function $P(v,t)$ of $V(t)$ and find a stationary solution, $P_{\mathrm{st}}(v)$.