線形制御理論

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図1で示す筒型の撹拌タンクを考える．左の管からは，定数濃度 c_1 [mol/m^3] の溶液が流量 q_1 [m^3/sec] で，右の管からは，定数濃度 c_2 [mol/m^3] の溶液が流量 q_2 [m^3/sec] で，それぞれタンクに流れ込んで，瞬時に撹拌され，一様な濃度 c [mol/m^3] をもつものとする．筒型タンクの底面積は A [m^2] であり，流出口の断面積は S [m^2] とする．流出口での流量 f [m^3/sec] はトリチェリの定理に従っており，$f = S\sqrt{2gh}$ を満たすものとする．ただし h [m] は液面位，g [m/sec^2] は重力加速度である．

流量を一定値 $q_1 = q_{1,0}$，$q_2 = q_{2,0}$ とするとき，液面位は $h = h_0$，タンク内濃度は $c = c_0$ で平衡状態にあるものとする．各定数は表1の値をとるものとして，以下の問いに答えよ．

(i) 平衡液面位 h_0 と平衡濃度 c_0 を求めよ．

(ii) 流量が $q_1 = q_{1,0} + u$，$q_2 = q_{2,0} + v$ と変化したとき，液面位が $h = h_0 + x$，濃度が $c = c_0 + y$ と変化するものとする．(i) で求めた平衡状態周りの線形近似モデルを考えるとき，u から x，u から y，v から x，および v から y への伝達関数をそれぞれ求めよ．

(iii) (ii) の線形近似モデルに対して，$u = -k_1 x, v = -k_2 y$ と線形フィードバック則を与えるとき，閉ループ系を安定にするゲイン (k_1, k_2) の集合を求めよ．

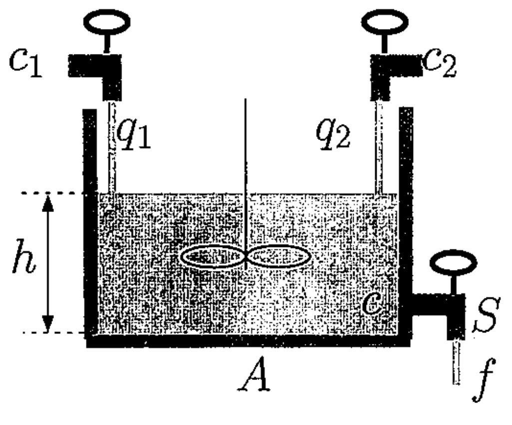

図1: 撹拌タンク

表1: 各定数の値

g	9.8
c_1	100
c_2	800
S	0.01
A	2
$q_{1,0}$	0.05
$q_{2,0}$	0.02

Linear Control Theory

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Fig. 1 shows a stirred tank. The tank is fed with two incoming flows with time-varying flow rates q_1 [m^3/sec] and q_2 [m^3/sec]. Both feeds contain dissolved material with constant concentrations c_1 and c_2. Assume that the material is stirred instantaneously and the time-varying concentration c [mol/m^3] of the solution in the tank is uniform. The cylinder-shaped tank has constant cross-sectional area A [m^2]. Assume further that the outgoing flow f [m^3/sec] obeys the Torricelli's theorem, namely, $f = S\sqrt{2gh}$, where S [m^2] is the cross-sectional area of the flow, g [m/sec^2] is the gravitational acceleration, and h [m] is the height of the liquid.

Let $h = h_0$ and $c = c_0$ be the height and concentration of the solution in equilibrium when the incoming flows $q_1 = q_{1,0}$ and $q_2 = q_{2,0}$ are constant. The values of the constants are given in Table 1. Answer the following questions.

(i) Calculate the height and the concentration of the solution in equilibrium.

(ii) When the constant incoming flows $q_{1,0}$ and $q_{2,0}$ are perturbed by u and v, *i.e.*, $q_1 = q_{1,0} + u$ and $q_2 = q_{2,0} + v$, the height and the concentration of the solution become $h = h_0 + x$ and $c = c_0 + y$, respectively. Consider the approximated linear model around the equilibrium point calculated in (i). Derive the transfer functions from u to x, u to y, v to x, and v to y.

(iii) Determine the set of stabilizing gains (k_1, k_2) when the linear feedback control law $u = -k_1 x$ and $v = -k_2 y$ is applied to the approximated linear model derived in (ii).

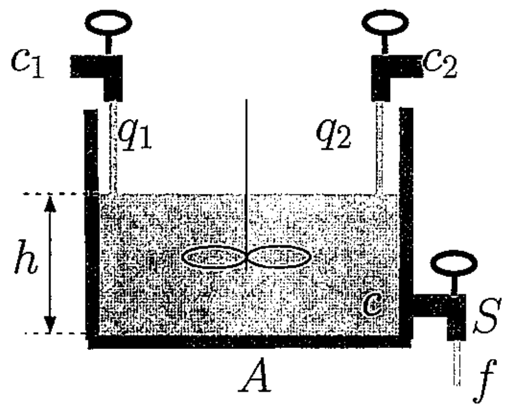

Fig.1: Stirred tank

Table 1: Constants

g	9.8
c_1	100
c_2	800
S	0.01
A	2
$q_{1,0}$	0.05
$q_{2,0}$	0.02