応用数学

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$R>0$ とし，図のように複素平面において，$C_1(R)$ を iR を始点とし 0 を終点とする線分，$C_2(R)$ を 0 を始点とし $(1+i)R$ を終点とする線分，$C_3(R)$ を $(1+i)R$ を始点とし iR を終点とする線分とする．$f(z)=e^{\frac{1}{2}z^2}$ $(z\in\mathbb{C})$ とおき，

$$A=\int_0^\infty \cos(t^2)dt, \quad B=\int_0^\infty \sin(t^2)dt$$

とおく．以下の問いに答えよ．なお，

$$\int_0^\infty e^{-\frac{1}{2}t^2}dt=\sqrt{\frac{\pi}{2}}$$

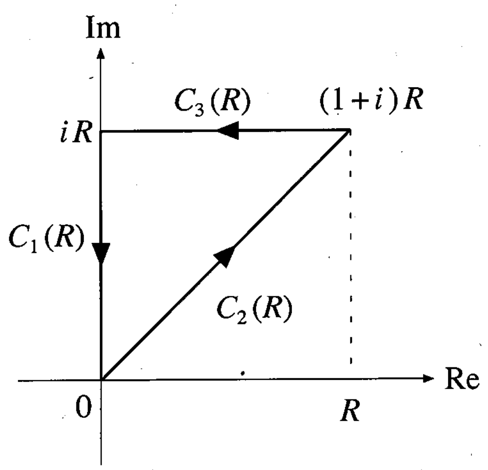
は用いてよい．

(i) $\displaystyle\lim_{R\to+\infty}\int_{C_1(R)} f(z)dz$ を求めよ．

(ii) $\displaystyle\lim_{R\to+\infty}\int_{C_2(R)} f(z)dz$ を A と B とを用いて表せ．

(iii) $\displaystyle\lim_{R\to+\infty}\int_{C_3(R)} f(z)dz=0$ を示せ．

(iv) A と B を求めよ．



An English Translation:

Applied Mathematics

$\boxed{1}$

Let $R > 0$ and let $C_1(R)$, $C_2(R)$ and $C_3(R)$ be the paths from iR to 0, from 0 to $(1+i)R$, and from $(1+i)R$ to iR, respectively, in the complex plane, as shown in the figure. Define $f(z) = e^{\frac{1}{2}z^2}$ $(z \in \mathbb{C})$ and let

$$A = \int_0^\infty \cos(t^2)dt, \quad B = \int_0^\infty \sin(t^2)dt.$$

Answer the following questions. Here you can use the equality

$$\int_0^\infty e^{-\frac{1}{2}t^2}dt = \sqrt{\frac{\pi}{2}}.$$

(i) Obtain $\lim_{R\to+\infty} \int_{C_1(R)} f(z)dz$.

(ii) Write $\lim_{R\to+\infty} \int_{C_2(R)} f(z)dz$ in terms of A and B.

(iii) Show that $\lim_{R\to+\infty} \int_{C_3(R)} f(z)dz = 0$.

(iv) Obtain A and B.