基礎力学

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ポテンシャル $V(r) = \dfrac{k}{r^n} \quad (k > 0, n \geqq 1)$ をもつ中心力による質量 m の粒子の散乱を考える．ここで，力の中心から粒子までの距離を r とし，r の最小値を r_0，力の中心のまわりの角運動量の大きさを $h(> 0)$ とし，無限遠方での粒子の速さを v_∞ とする．以下の問いに答えよ．

(i) $r = r_0$ の時の粒子の速さ v_0 を求めよ．

(ii) 散乱角 Θ が

$$\Theta = \pi - 2\int_0^{u_0} \frac{du}{\sqrt{u_0^2 - u^2 + \frac{2m}{h^2}[V(\frac{1}{u_0}) - V(\frac{1}{u})]}}$$

で与えられることを示せ．但し，$u = \dfrac{1}{r}$, $u_0 = \dfrac{1}{r_0}$ とする．

(iii) ポテンシャルが $V(r) = \dfrac{k}{r} \quad (k > 0)$ で与えられる場合の散乱の微分断面積を導出せよ．

An English Translation:

Basic Mechanics

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Let us consider a particle of mass m scattering under the action of a central force by a potential $V(r) = \dfrac{k}{r^n}$ $(k > 0, n \geqq 1)$ where r denotes the distance between the particle and the center of the central force. Let r_0 be the minimal value of r, $h(> 0)$ be the magnitude of the angular moment around the center of the central force and v_∞ be the speed of the particle at $r = \infty$. Answer the following questions.

(i) Obtain the speed v_0 of the particle at $r = r_0$.

(ii) Show that the scattering angle Θ is given by

$$\Theta = \pi - 2\int_0^{u_0} \frac{du}{\sqrt{u_0^2 - u^2 + \frac{2m}{h^2}[V(\frac{1}{u_0}) - V(\frac{1}{u})]}}$$

where $u = \dfrac{1}{r}$ and $u_0 = \dfrac{1}{r_0}$.

(iii) Derive the scattering differential cross section in the case that $V(r) = \dfrac{k}{r}$ where k is a positive constant.