応用数学

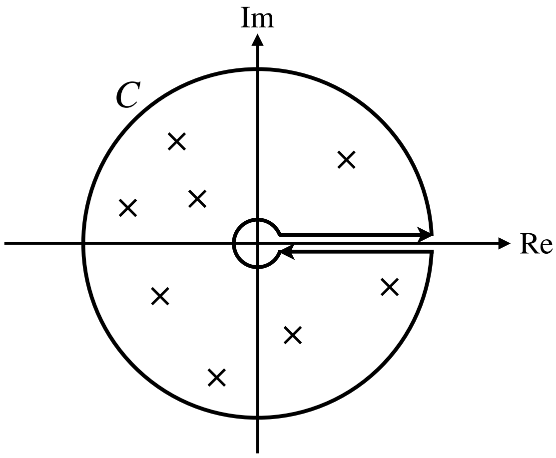
1

複素数平面上の有理関数 $f(z)$ の極を $a_1, a_2, \ldots, a_K$ とし，非負の実軸上にないものとする．ここで，関数 $f(z)$ は $|z| \to \infty$ で $|z^2 f(z)| \to 0$ をみたすとする．点 $z = a$ における関数 $g(z)$ の留数を $\operatorname*{Res}_{z=a}(g(z))$ で表す．以下の問いに答えよ．

(i) z の偏角の範囲を $I = [0, 2\pi)$ とすることで，$\log z$ の枝を定め，C を図のように極 $a_1, a_2, \ldots, a_K$ を囲む閉曲線とする．このとき，$\oint_C f(z) \log z \, dz$ を $\operatorname*{Res}_{z=a_k}(f(z)\log z)\,(k = 1, 2, \ldots, K)$ を用いて表わせ．

(ii) $\int_0^\infty f(x) dx$ を $\operatorname*{Res}_{z=a_k}(f(z)\log z)\,(k = 1, 2, \ldots, K)$ を用いて表わせ．

(iii) $\displaystyle\int_0^\infty \frac{dx}{(x+1)(x^4+1)}$ を求めよ．



An English Translation:

Applied Mathematics

1

Let $a_1, a_2, \ldots, a_K$ be the poles of the rational function $f(z)$ on the complex plane. These points $a_1, a_2, \ldots, a_K$ do not lie on the non-negative real axis. The function $f(z)$ satisfies $|z^2 f(z)| \to 0$ as $|z| \to \infty$. Let $\operatorname*{Res}_{z=a}(g(z))$ denote the residue of a function $g(z)$ at a point $z = a$. Answer the following questions.

(i) Let us define the branch of $\log z$ by fixing the range $I = [0, 2\pi)$ of the argument of z, and let C be a closed curve enclosing $a_1, a_2, \ldots, a_K$ as shown in the figure. Write $\oint_C f(z) \log z \, dz$ in terms of $\operatorname*{Res}_{z=a_k}(f(z) \log z)$ $(k = 1, 2, \ldots, K)$.

(ii) Write $\int_0^\infty f(x)dx$ in terms of $\operatorname*{Res}_{z=a_k}(f(z) \log z)$ $(k = 1, 2, \ldots, K)$.

(iii) Obtain $\displaystyle\int_0^\infty \frac{dx}{(x+1)(x^4+1)}$.