

## アルゴリズム基礎

2

$G = (V, E)$  を点集合  $V$ , 枝集合  $E$  から成る単純有向グラフとする.  $R(u; G)$  を  $G$  において点  $u$  から有向路で到達できる点の集合と定め,  $\text{dist}(u, v; G)$  を点  $u$  から点  $v$  へ至る  $G$  の有向路の最短の長さとする.  $v \notin R(u; G)$  のときは  $\text{dist}(u, v; G) \triangleq |V|$  と定める. 有向グラフ  $G$  から有向枝  $e \in E$  を削除した有向グラフを  $G - e$  と記す.  $s, t$  を  $V$  の二点とする.  $G$  は隣接リストにより貯えられているとする. 以下の問いに答えよ.

- (i)  $t \in R(s; G)$  と仮定する. 点  $s$  から点  $t$  へ至る有向路で最短のものを求める  $O(|V| + |E|)$  時間アルゴリズムを与えよ.
- (ii)  $\text{dist}(s, t; G - e) > \text{dist}(s, t; G)$  を満たす有向枝  $e \in E$  が存在するかどうかを判定する  $O(|V| + |E|)$  時間アルゴリズムを与えよ.
- (iii)  $\text{dist}(s, t; G) = \text{dist}(t, s; G) = 3 < \text{dist}(s, t; G - e) = \text{dist}(t, s; G - e)$  である二点  $s, t \in V$ , 有向枝  $e \in E$  をもつ有向グラフ  $G = (V, E)$  の例を作成せよ.

An English Translation:

## Data Structures and Algorithms

2

Let  $G = (V, E)$  be a simple directed graph with a vertex set  $V$  and an edge set  $E$ . Let  $R(u; G)$  denote the set of vertices reachable from a vertex  $u$  by a directed path in  $G$  and  $\text{dist}(u, v; G)$  denote the shortest length of a path from a vertex  $u$  to a vertex  $v$  in  $G$ , where we set  $\text{dist}(u, v; G) \triangleq |V|$  if  $v \notin R(u; G)$ . Let  $G - e$  denote the directed graph obtained from  $G$  by removing a directed edge  $e \in E$ . Let  $s$  and  $t$  be two vertices in  $V$ . Assume that  $G$  is stored in adjacency lists. Answer the following questions.

- (i) Assume that  $t \in R(s; G)$ . Give an  $O(|V| + |E|)$ -time algorithm that computes a directed path with the shortest length from  $s$  to  $t$ .
- (ii) Give an  $O(|V| + |E|)$ -time algorithm that tests whether there exists a directed edge  $e \in E$  such that  $\text{dist}(s, t; G - e) > \text{dist}(s, t; G)$ .
- (iii) Construct an example of a directed graph  $G = (V, E)$  that contains two vertices  $s, t \in V$  and a directed edge  $e \in E$  such that  $\text{dist}(s, t; G) = \text{dist}(t, s; G) = 3 < \text{dist}(s, t; G - e) = \text{dist}(t, s; G - e)$ .