

力学系数学

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n を自然数, ij 成分が

$$a_{ij}(t) = \begin{cases} 1 & (i = j \text{ のとき}); \\ t & (i = j + 1 \text{ のとき}); \\ 0 & (\text{上記以外}) \end{cases}$$

の n 次正方行列を $A(t)$ として, $t > 0$ において n 元連立線形微分方程式

$$\frac{dx}{dt} = \frac{1}{t}A(t)x, \quad x \in \mathbb{R}^n$$

を考える. 以下の問いに答えよ.

- (i) $n = 1$ のとき一般解を求めよ.
- (ii) $n = 2$ のとき一般解を求めよ.
- (iii) 任意の自然数 n に対して一般解を求めよ.

An English Translation:

Mathematics for Dynamical Systems

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Let n be a positive integer and let $A(t)$ be an $n \times n$ matrix whose ij -component is given by

$$a_{ij}(t) = \begin{cases} 1 & (\text{for } i = j); \\ t & (\text{for } i = j + 1); \\ 0 & (\text{otherwise}). \end{cases}$$

Consider the n -dimensional system of differential equations

$$\frac{dx}{dt} = \frac{1}{t}A(t)x, \quad x \in \mathbb{R}^n.$$

Here $t > 0$. Answer the following questions.

- (i) Obtain a general solution when $n = 1$.
- (ii) Obtain a general solution when $n = 2$.
- (iii) Obtain a general solution when n is an arbitrary positive integer.