基礎数学 I

1

区間 $[0,1]$ 上の C^1 級関数 $\varphi(t)$ で $\varphi(0)=0, \varphi(1)=1$ を満たすもの全体の集合を Γ とする．$\varphi \in \Gamma$ に対して，

$$I(\varphi) = \int_0^1 (\dot{\varphi}(t))^2 dt, \qquad J(\varphi) = \int_0^1 (t\dot{\varphi}(t))^2 dt$$

とおく．ここで，$\dot{\varphi}(t)$ は $\varphi(t)$ の導関数 $\dfrac{d\varphi}{dt}(t)$ である．また，

$$A = \inf_{\varphi \in \Gamma} I(\varphi), \qquad B = \inf_{\varphi \in \Gamma} J(\varphi)$$

とおく．以下の問いに答えよ．

(i) A を求めよ．また，$I(\varphi) = A$ となる $\varphi \in \Gamma$ を求めよ．

(ii) 正の整数 n に対して，$\varphi_n(t) = 1 - (1-t)^n$ とおく．$J(\varphi_n)$ を求めよ．

(iii) B を求めよ．

(iv) $J(\varphi) = B$ となる $\varphi \in \Gamma$ は存在しないことを示せ．

An English Translation:

Basic Mathematics I

1

Let Γ be the set of C^1 functions $\varphi(t)$ on the interval $[0,1]$ satisfying $\varphi(0) = 0$ and $\varphi(1) = 1$. For $\varphi \in \Gamma$, define

$$I(\varphi) = \int_0^1 (\dot{\varphi}(t))^2 dt, \qquad J(\varphi) = \int_0^1 (t\dot{\varphi}(t))^2 dt,$$

where $\dot{\varphi}(t)$ is the derivative $\dfrac{d\varphi}{dt}(t)$ of $\varphi(t)$. Let

$$A = \inf_{\varphi\in\Gamma} I(\varphi), \qquad B = \inf_{\varphi\in\Gamma} J(\varphi).$$

Answer the following questions.

(i) Find A. Moreover, find $\varphi \in \Gamma$ satisfying $I(\varphi) = A$.

(ii) For a positive integer n, define $\varphi_n(t) = 1 - (1-t)^n$. Find $J(\varphi_n)$.

(iii) Find B.

(iv) Show that there does not exist $\varphi \in \Gamma$ satisfying $J(\varphi) = B$.