常微分方程式

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$a, b, c, d \in \mathbb{R}$ を定数として次の微分方程式を考える.

$$t^2 \frac{dx}{dt} + (at + b)x = ct + d \tag{1}$$

以下の問いに答えよ．ただし，$b \neq 0$ とし，自然数 n に対して最高次の次数が n の t の多項式で表される解を n 次多項式解と呼ぶ.

(i) 式 (1) が 1 次多項式解をもつための必要十分条件を a, b, c, d を用いて表わせ.

(ii) 自然数 $n > 1$ に対して，式 (1) が n 次多項式解をもつための必要十分条件を a, b, c, d, n を用いて表わせ.

(iii) どんな自然数 n に対しても式 (1) が n 次多項式解をもたないための必要十分条件を a, b, c, d を用いて表わせ.

An English Translation:

Ordinary Differential Equations

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Let $a, b, c, d \in \mathbb{R}$ be constants and consider the differential equation

$$t^2 \frac{dx}{dt} + (at + b)x = ct + d, \tag{1}$$

where $b \neq 0$. For a positive integer n, a solution is called an nth-order polynomial solution if it is an nth-order polynomial of t containing a nonzero nth-order term. Answer the following questions.

(i) Obtain a necessary and sufficient condition for equation (1) to have a first-order polynomial solution, and express the condition with a, b, c and d.

(ii) Let $n > 1$ be an integer. Obtain a necessary and sufficient condition for equation (1) to have an nth-order polynomial solution, and express the condition with a, b, c, d and n.

(iii) Obtain a necessary and sufficient condition for equation (1) to have no nth-order polynomial solution for any positive integer n, and express the condition with a, b, c and d.