

線形代数

2

以下の問いに答えよ.

(i) ベクトル

$$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} a \\ 0 \\ 1 \end{pmatrix}$$

が一次独立となる a の条件を求めよ

(ii) 次式を計算せよ.

$$\det \begin{pmatrix} -2 & 1 & 3 \\ 4 & -3 & -5 \\ -5 & 3 & 8 \end{pmatrix}$$

(iii) 次の行列の逆行列を求めよ.

$$\begin{pmatrix} -3 & 2 & 2 \\ -2 & 2 & 1 \\ 2 & -1 & -1 \end{pmatrix}$$

(iv) 線形空間 $\{\mathbf{x} \in \mathbb{R}^3 \mid \mathbf{x} \cdot \mathbf{a} = 0\}$ の基底と次元を求めよ. ただし,

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

で, $\mathbf{x} \cdot \mathbf{a}$ は \mathbf{x} と \mathbf{a} の内積である.

(v) 行列

$$A = \begin{pmatrix} -4 & -3 \\ 6 & 5 \end{pmatrix}$$

を対角化せよ.

An English Translation:

Linear Algebra

2

Answer the following equations.

(i) Find the condition on a that vectors

$$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} a \\ 0 \\ 1 \end{pmatrix}$$

are linearly independent.

(ii) Compute

$$\det \begin{pmatrix} -2 & 1 & 3 \\ 4 & -3 & -5 \\ -5 & 3 & 8 \end{pmatrix}.$$

(iii) Find the inverse matrix of

$$\begin{pmatrix} -3 & 2 & 2 \\ -2 & 2 & 1 \\ 2 & -1 & -1 \end{pmatrix}.$$

(iv) Find a basis and the dimension of the linear space $\{\mathbf{x} \in \mathbb{R}^3 \mid \mathbf{x} \cdot \mathbf{a} = 0\}$ where

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix},$$

and $\mathbf{x} \cdot \mathbf{a}$ is the inner product of \mathbf{x} and \mathbf{a} .

(v) Diagonalize

$$A = \begin{pmatrix} -4 & -3 \\ 6 & 5 \end{pmatrix}.$$