微積分

$\boxed{1}$

以下の問いに答えよ.

(i) $\sin x = 1 - x^2$ を満たす実数 x が存在することを示せ.

(ii) 関数

$$\frac{3x}{x^2 - x - 2}$$

の原始関数を求めよ.

(iii) $z = f(x, y)$ を C^1 級関数とし, $x = \dfrac{u^2 - v^2}{2}, y = uv$ とする. $\dfrac{\partial z}{\partial u}, \dfrac{\partial z}{\partial v}$ を $u, v, \dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}$ を用いて表せ.

(iv) $f(x, y) = x^3 + y^3 - 3xy$ のすべての極値を求めよ.

(v) $D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leqq y \leqq x \leqq 1\}$ とする.

$$\iint_D xe^y dxdy$$

を求めよ.

An English Translation:

Calculus

$\boxed{1}$

Answer the following questions.

(i) Prove that there exists a real number x satisfying $\sin x = 1 - x^2$.

(ii) Find the primitive function of

$$\frac{3x}{x^2 - x - 2}.$$

(iii) Let $z = f(x, y)$ be a C^1-class function, and let $x = \frac{u^2 - v^2}{2}$ and $y = uv$. Express $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ by using u, v, $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(iv) Find all the extrema of $f(x, y) = x^3 + y^3 - 3xy$.

(v) Let $D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leqq y \leqq x \leqq 1\}$. Find

$$\iint_D xe^y dxdy.$$